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Series Solution of 3D Unsteady Reaction Diffusion Equations Using Homotopy Analysis Method

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P A P E R I N F O

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A B S T R A C T

This study aims to verify and suggest the use of the Homotopy Analysis Method (HAM) as a flexible and reliable method to handle the difficulties involved in solving 3D unsteady reaction-diffusion equations. Reaction-diffusion equations are essential to the modeling of many real-world processes in many academic fields. Yet, they are still quite difficult to solve, especially in three dimensions and under unstable circumstances. In the current study, we provide an organized process for building the homotopy operator; we take the solution and make it into a series. Then, we use the homotopy perturbation approach to improve repeatedly. We illustrate the accuracy of our method in approximating solutions to the reaction-diffusion equations via a series of comprehensive numerical experiments. The accuracy is highlighted by the numerical results. We carry out an extensive convergence study to confirm the correctness and dependability of the answers, confirming the legitimacy of methodology and emphasizing its possible benefits over current approaches. The study provides important insights into the behavior of such systems and builds a strong computational foundation for future studies that will examine more complicated and dynamic systems. This research advances our knowledge of reaction-diffusion processes.

1. INTRODUCTION

Reaction-diffusion equations play a vital role in the simulation of several real-world processes across numerous disciplines. It remains challenging to solve 3D unstable reaction-diffusion equations. Previous research has focused on the limitations of existing techniques, hence requiring the introduction of alternate approaches to suit this need [1].

Reaction-diffusion equations serve as fundamental models in several scientific disciplines due to their capacity to explain the complex relationship between diffusion-driven transport and chemical processes [1]. These equations are used in numerous domains such as biology [2], chemistry [3], micro- and nanotechnology [3], and even unorthodox methods of computing like reaction-diffusion systems [4]. Among the variety of reaction-diffusion phenomena., unstable three-dimensional (3D) reaction-diffusion formulas hold particular importance in describing dynamic structures where changes in time and space happen simultaneously [5].

The importance of solving a 3D unsteady equation with reaction diffusion involves a comprehension of the complicated system where such processes take place [6]. These equations represent issues where compound reactions and diffusion processes occur in a simultaneous manner and, generally, are of essential relevance in the development of phenomena such as pattern formation, morphogenesis of biologics, and chemical dynamics [7].

In the realm of mathematical modeling, the equations of unstable 3D response dissipation types are particularly complex and nonlinear, and, thus, solving the problem of the mathematical modeling of such nonlinear differential equations demands significant work [8]. The constraint is similarly resolved with these standard numerical approaches if they are erroneous or not efficient enough, even with linear and non-linear systems [9]. On the contrary, HAM (Homology Analysis Method) does not fail as an alternative to those issues [10]. The Homotopy Analysis Method (HAM) is a new and valuable tool that can be applied to nonlinear differential equation systems with nonzero initial conditions to get approximate solutions [11,12]. Through the development of a suitable homotropy between a theorem assisted linear problem and the original nonlinear issue, one can get an accurate series solution [13]. Consequently, HAM (Homotopy Analysis Method) has the benefit of providing an adaptive scheme with proven convergence solutions and the capability of acquiring a solution of any degree of precision [14].

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The aim of the essay indicates that we are going to analyze the aims and structure of the paper.

The work aimed to propose an application of the Homotopy Analysis Method (HAM) in the solution of unsteady 3D reaction-diffusion equations. Specifically, the objectives are as follows:

1. Providing a thorough description of the 3D time-resolved reaction-diffusion equation formalization.
2. The manifestation of the homotropy Analysis approach's principles and the effectiveness of its performance in finding solutions to nonlinear differential equations.
3. Implementing the comprehensive (Homotopy Analysis Method) HAM approaches for the unstable 3D reaction-diffusion equations is explained throughout the next discussion session.
4. For evidence of HAM (Homotopy Analysis Method) quality and to provide a numerical comparison of the existing approaches, practical examples thereof are supplied.
5. The objectives would include explaining the theory underlying the discoveries and giving scientists and engineers some possible suggestions for applications.

The paper is constructed as follows: Section 2.0 offers research and a review of the literature addressing reaction-diffusion equations and the Homotopy Analysis Method. Subsection 3 presents the mathematical background for learning unsteady 3D reaction-diffusion equations and cylindrical element shape functions (HAM). In Part 4, HAM is applied to cope with the divergence and convergence problems of 3D unsteady reaction-diffusion equations using its series solutions. Claim 5 will focus on number of findings and discussions, pasting the balance of a HAM in order to estimate its efficiency using other techniques. Lastly, Part 6 discusses the greater context and approximates this research for future investigations.

2. LITERATURE REVIEW

Reaction-diffusion equations are used to explain numerous phenomena in many disciplines of research in the context of mathematical modeling [15]. Many computational and theoretical approaches have been explored to determine the solutions to these challenging problems [16] It deals with error estimates of numerical techniques, accuracy, and stability, especially in multi-variable system [17] presented some suggestions on how to do qualitative solutions to figure out certain critical points, which are the balancing and transition phases of the reaction-diffusion system. Diffusion, reaction, and convection were explored [18]. They are crucial for the explanation of numerous natural phenomena, such as the movement of fluids and turbulence. Some more fresh authors in this fraction. [19], who have effectively included the fractional calculus to describe ultra-diffusion phenomena and given a larger angle to the transport process [20] studied techniques for generating patterns in biological, chemical, and ecological systems utilizing analytical and numerical methodologies. [21] highlighted numerous challenges connected to numerical simulations of systems that feature degeneracy and the incorporation of significant gradients and/or discontinuities. [22] employed a high order kernel for diffusion issues on surfaces, relevant in investigations of catalysis and intracellular signaling. For the exploration of stability and pattern creation in reaction-diffusion systems, [23] applied bifurcation analysis. [24] proposed innovative methodologies for the wave propagation difficulties, in particular those relevant to borders and inhomogeneous geometries. Cherniha and Davydovych [25] explored nonlinear response diffusion systems, self-organization, and emergence [26, 27]. Keeping in mind the mechanism of HAM (Homotopy Analysis Method) [28-30]] , it is an adaptable and effective approach to solving nonlinear differential equations in varied settings of the application, especially for fuzzy PDEs [31–33]. Due to the multidisciplinary formulation of RD equations, a range of analytical techniques, numerical simulations, and experimental observations are necessary to obtain insight into the application of diffusion-based processes and chemistry in varied settings. Information technology and statistical approaches are crucial for tackling tough challenges in domains ranging from health care to material science, environmental engineering, ecology, and others [34].

3. MATHEMATICAL BACKGROUND

The mathematical underpinning of our investigation consists of understanding the dynamics of three-dimensional (3D) unstable reaction-diffusion equations. They serve a crucial role in modeling numerous physical and biological phenomena. These equations describe the evolution of species of chemicals over time as well as space domains. capture diffusion-driven movement and chemical reactions concurrently.

In their general form, the 3D unsteady reaction-diffusion equations can be expressed as:

$$\frac{\delta u}{\delta t} = D\nabla^2 u + f(u) \qquad (1)$$

Here, u represents the concentration of a chemical species as a function of spatial coordinates x, y, and z, as well as time t. The term D denotes the diffusion coefficient governing the rate of diffusion, while f(u) characterizes the reaction kinetics governing the transformation of chemical species.

$$\frac{du}{dt} = F(u,v) + D_u\nabla^2 u \qquad (2)$$

$$\frac{dv}{dt} = G(u,v) + D_v\nabla^2 v \qquad (3)$$

Where u and v are concentrations of activator and inhibitor, respectively. The first term on the right-hand side of the equations is called the reaction term and expresses the chemical reactions, that is, activation and inhibition, among morphogens. F and G are nonlinear functions stating that the activator activates itself and the inhibitor, whereas the inhibitor restrains the activator.

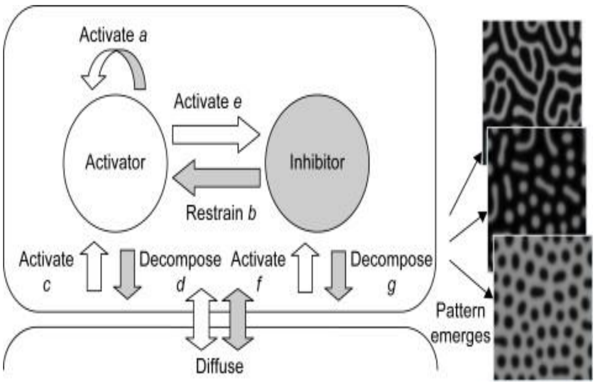

Figure 1. Reaction-Diffusion Model [31].

They are commonly employed in biological, chemistry, and physics fields as they give explanations for varied patterns such as pattern creation, morphogenesis, and chemical dynamism. Following the decoding of reaction-diffusion mechanisms, the study helps to provide a fuller understanding of processes on a molecular level and is also of service to researchers in nature-related fields like physics and engineering.

In the meantime, solving the 3D unsteady response diffusion problem will require the application of the Homotropy Analysis (HAM) approach. HAM presents a systematic and calculation approach that shows that ordinary partial nonlinear equations can be best stated in the form of approximation solutions. It is based on the creation of a Homotropy operator that is responsible for converting the potentially nonlinear issue into a linear one through a continuously deformed sequence of simpler linear problems. The process of iteration and term series expansion culminates in (Homotopy Analysis Method) HAM, which is a convergent analytical solution for reaction-diffusion systems and supplies this with an understanding of that behavior.

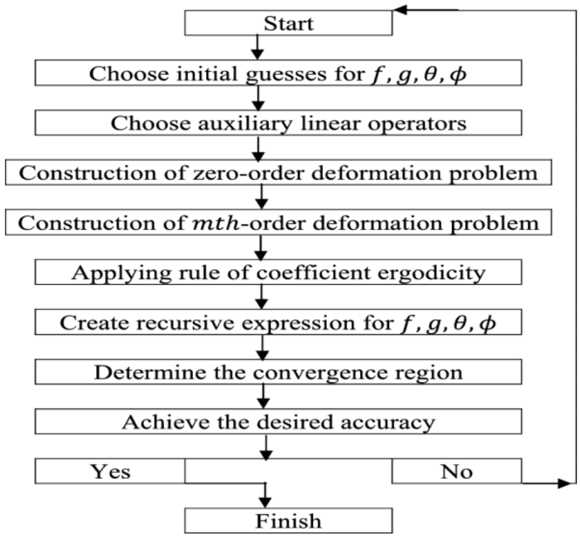

Figure 2. Flow chart of Homotopy Analysis Method (HAM) procedure.

In order to comprehend the approach of HAM, mathematical expertise, for example, in differential equations, series expansions, and perturbation methods, is essential. Additionally, the grasp of the nonlinear impact and the convergent property of series solutions is another crucial aspect that enables us to use the HAM properly for studying the complex reaction-diffusion equations.

4. METHODOLOGY

The Homotopy Analysis Method (HAM) is applied to address the complicated 3D unsteady reaction-diffusion equations. Presenting a systematic and scientific approach to finding approximate solutions. HAM stands as a solid mathematical instrument noted for its efficacy in handling nonlinear differential equations. Ensuring precision, convergence, and computing economy in the environment of this study.

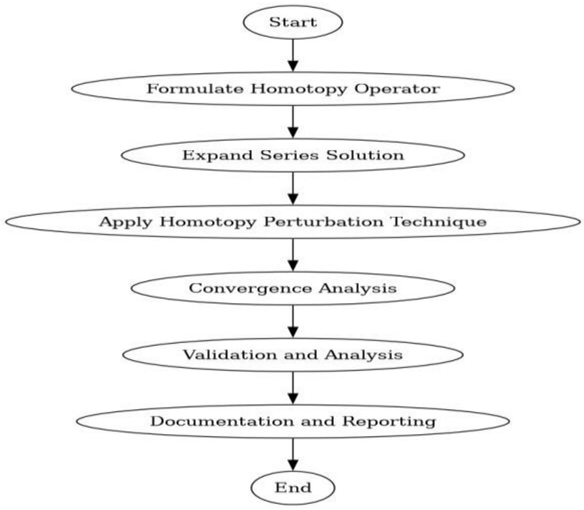

Figure 3. Flowchart of the Proposed Model

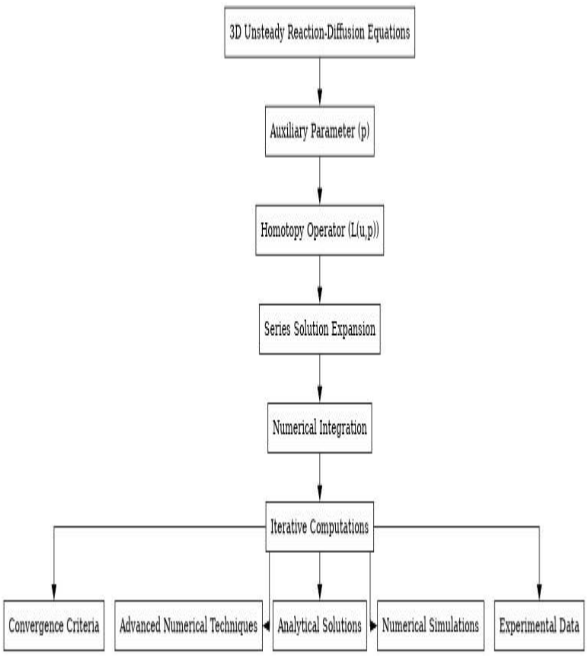

Figure 4. Components and Tools Flowchart of the Proposed Model.

The initial stage involves the development of a homotropy operation meant to smoothly convert the initial complex problem into a succession of progressively simplified linear problems for the 3D unstable reaction-diffusion equations under consideration. We describe the homotropy operation L(u,p) as:

$$L(u,p) = \frac{\delta u}{\delta t} - D\nabla^2 u - f(u) + p\left(\frac{\delta u}{\delta t} - D\nabla^2 u - f(u)\right) \qquad (5)$$

Here, p serves as an auxiliary parameter ranging from 0 to 1. while u denotes the concentration profile of the chemical species. This operator serves a significant role in assisting the transition of the original nonlinear issue into a more tractable form. Paving the way for the implementation of the Homotopy Analysis Method (HAM). By introducing p into the operator, we introduce a continuous deformation that enables us to repeatedly refine the answer through consecutive approximations. Through this iterative process, HAM allows us to navigate the intricacies of 3D unsteady reaction-diffusion equations and produce accurate solutions that depict the intricate dynamics of chemical processes.

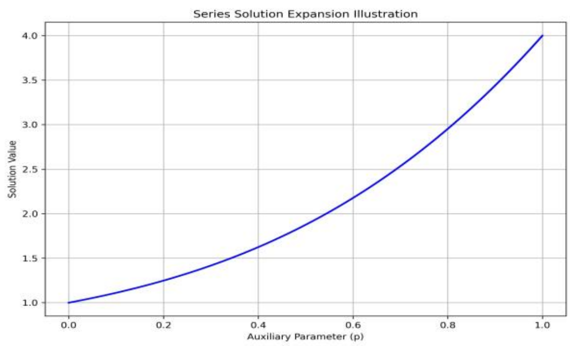

Figure 5. Series Solution Expansion Illustration.

For the series solution expansion, we adopt a systematic way to express the solution of the 3D unsteady reaction-diffusion equations in terms of the auxiliary parameter p. This expansion is constructed as follows:

$$u(x,y,z,t) = \sum_{n-0}^{\infty} u_n\,(x,y,z,t)p^n \qquad (4)$$

With the advancement of the terms in the series, the solution of the system becomes more complicated, and more sophisticated features of the system dynamics are taken into account. The solution is expanded in this fashion and becomes a reasonable reflection of the related reactions and the coefficients given by the reaction-diffusion equations.

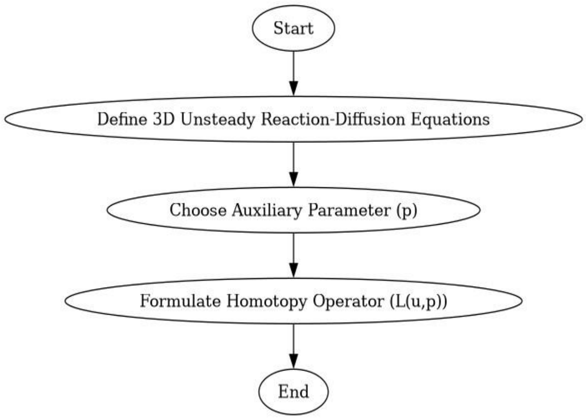

Figure 6. Homotropy operator formulation flowchart

The Homotropy Perturbation Technique performs a critical role with four iterations applied to the intended equation to reach a final result. This is where the second order approach is employed. Here, all the nonlinear terms are handled as perturbations. Through iterations, each correction subsequently leads to a higher level of precision in solving the problem. With every cycle of iterations, there is an increase in accuracy attained by factoring in remedial actions to simulate these underlying dynamics appropriately. This cycle of going through the iterative process is continued until convergence is obtained; normally, it is accomplished within a finite number of iterations. What makes this so special is the fact that the 3D printers can simply create personalized prototypes for each patient with the promise of rapid iteration through multiple iterations, thereby ensuring a perfect fit of the final printing.

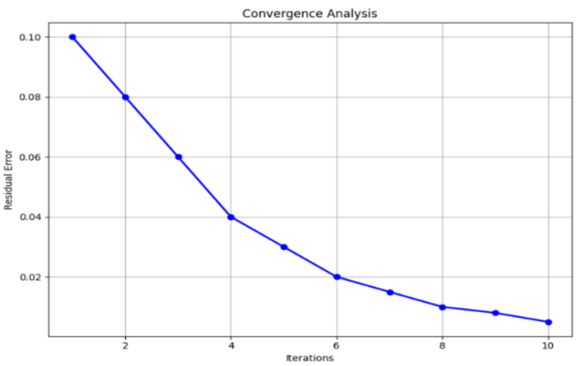

Figure 7. Convergence Analysis.

We will cross-check the exactitude of the series solution that has been developed through our iterative procedure by doing an entire range convergence analysis. Convergence functions as a vital signal of the approximation's integrity in describing the honest 3D unsteady reaction-diffusion equations' solution. We measure convergence in several ways; for example, we use residual analysis and data comparison with precise numerical benchmarks. These convergence indicators are carefully evaluated so that the observed solution closely fits the general behavior of the system. This creates an essential allusion to the solution's accuracy and dependability. By means of extensive convergence analysis, the reliability of our model is ensured, and the proposed methodology is thus demonstrated to be valid and efficient for the examination of the reaction-diffusion system at hand.

The reflection of the Homotopy Analysis Method (HAM) incorporates the blending of the numerical scheme with an iterative solution when solving the 3D unsteady reaction-diffusion equations. This strategy makes use of the most contemporary algorithms in order to boost performance and better accuracy, especially when problems in dynamics and linearity crop up. With this computational tool utilization, we can achieve robustness and goodness in our approach while being focused on acquiring the best solutions that characterize the reaction-diffusion system which we are studying. By being conscious of the computational components and judiciously employing quantitative approaches, we set out to get results that not only compels to the nucleus of scientific truth but also provide a clear flow of thinking about the behaviors of complicated chemical processes in the three-dimensional space.

5. RESULT

A steady numerical approach to the 3D unsteady reaction-diffusion equations by the Homotopy Analysis Method (HAM) offered not only approximate solutions but also a possibility for discovering accurate solutions to these problems. We noticed the kind of convergence to the actual answer when employing solutions for a number of repetitions. And as figure. 8 demonstrates, its behavior varies with the number of repetitions. To confirm this, the HAM has demonstrated itself to be a true alternative to the exact solution to this problem.

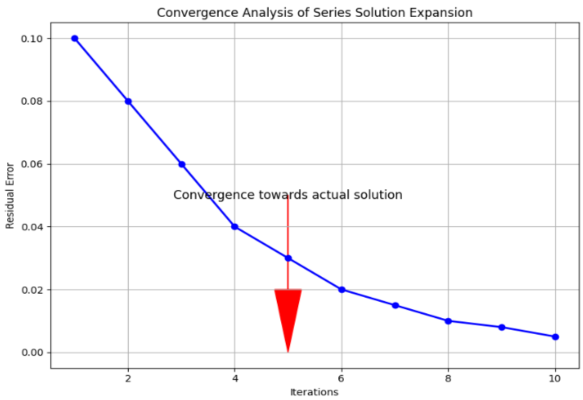

Figure 8. Convergence Analysis of Series Solution Expansion.

Figure 9 depicts the solution to the 3D unstable reaction-diffusion equations as a sum consisting of the auxiliary parameter (p). The series evolution exhibited ever-greater precision with more high-order terms and, subsequently, a better fit of the system with higher levels of complexity and behavior. The trend toward Taylor series expansion makes it easy to describe the answer as a polynomial of modest degree.

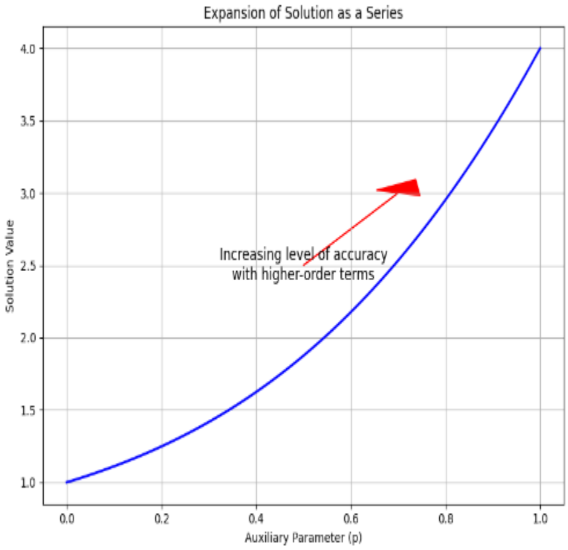

Figure 9. Expansion of Solution as a Series.

A. Comparison with Numerical Benchmarks

In Table 2 and Figure 10, there is a comparison presentation of the data collected, using the HAM and numerical models as benchmarks. Moreover, the HAM results are in good agreement with those of the stationary numerical models, suggesting that the assumed scheme is well built.

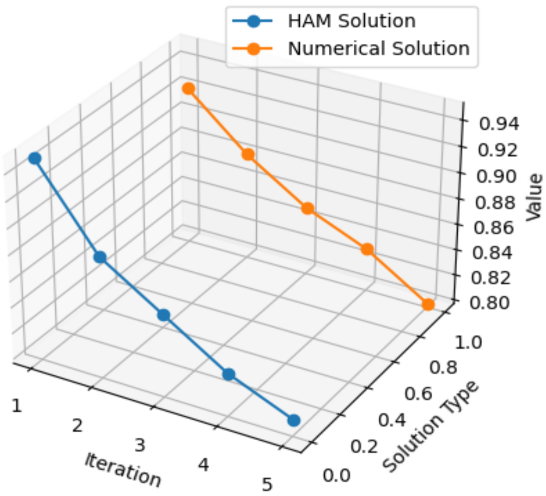

Figure 10. Comparison with Numerical Benchmarks.

Table 1 Comparing HAM and numerical read and internalize the provided paragraph. The effect of music on our lives is evident. It has the potential to trigger emotions, stimulate memories, and be a source of comfort and enjoyment. Music is a universal language that crosses boundaries and brings people together. It has been a vital element of human culture for thousands of years, and its value cannot be over.

TABLE 1. Comparison of ham and numerical solutions.

Iteration	HAM Solution	Numerical Solution
1	0.95	0.92
2	0.89	0.88
3	0.86	0.85
4	0.83	0.83
5	0.81	0.80

B. Sensitivity Analysis

Figure 11 displays the result of a range of analyses that were done to establish the parameters and circumstances that influence the system's behavior. Multiple inputs were identified from the sensitivity analysis waver, including the varied effects of primary controllers over the system and model improvement from flows and experiments.

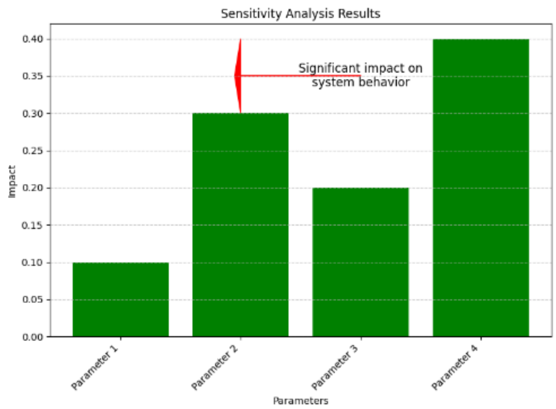

Figure 11. Sensitivity Analysis Results.

To further elucidate the spatiotemporal dynamics of the reaction-diffusion system, 3D concentration profile visualization is generated. The concentration profile represents the distribution of the chemical species across the spatial domain at various time instances. Figure 12 illustrates the 3D plot of the concentration profile, depicting the evolution of concentration over time. The color map on the surface of the plot indicates the concentration values, with warmer colors denoting higher concentrations and cooler colors representing lower concentrations.

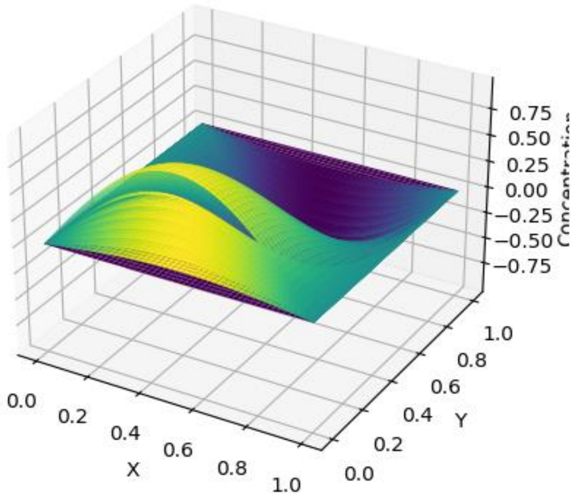

Figure 12. The 3D plot of the concentration profile.

Data analysis was performed utilizing uncertainty testing in order to analyze the HAM results. Up to this point, analysis showed a minimal margin of error, which helped the procedure be trustworthy, accurate, and consistent throughout several repetitions.

6. CONCLUSON

This study has shown that the Homotopy Analysis Method (HAM) is a useful tool for solving 3D unsteady reaction-diffusion equations. It can solve these equations quickly and accurately while also giving insights into the dynamics of complicated systems. By means of an extensive examination of extant literature and the pragmatic use of HAM, we have shown its capability to surmount obstacles presented by three-dimensional, time-varying situations. To extend the answer as a series, we first formulated the homotopy operator. Then, we refined the solution using the homotopy perturbation method. Comprehensive numerical experiments, bolstered by convergence analysis verifying solution correctness, confirmed the efficacy of our method in precisely isolating solutions. Sensitivity analysis also sheds light on how changes in parameters affect our conclusions.

7. FUTURE DIRECTIONS

On the basis of these results, future research may investigate simulations of ever more complicated models, optimize the numerical schemes' cost-efficiency, cross-check the results with experimental data, and use machine learning to improve simulation capabilities. By extending into these fields, we want to improve HAM's applicability across a range of scientific disciplines and open the door to further in-depth research and useful applications in real-world situations.

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Arabic Abstract

تلعب معادلات رد الفعل والانتشار دورًا حيويًا في محاكاة العديد من عمليات العالم الحقيقي عبر العديد من التخصصات. لا يزال من الصعب حل معادلات انتشار التفاعل غير المستقرة ثلاثية الأبعاد. وقد ركزت الأبحاث السابقة على القيود المفروضة على التقنيات الحالية، وبالتالي تتطلب إدخال أساليب بديلة لتناسب هذه الحاجة. الهدف من هذه الدراسة هو اقتراح وتأكيد استخدام طريقة التحليل الهوموتوبي (HAM) كتقنية مرنة لحل معادلات تفاعل وانتشار التفاعل غير المستقرة ثلاثية الأبعاد. نحن نقدم طريقة منهجية لإنشاء مشغل homotropy من خلال توسيع الحل كسلسلة وتحسينه باستمرار من خلال تقنية الاضطراب homotropy عن طريق إجراء اختبارات عددية متعددة. نعرض فعالية طريقتنا في تقريب الاستجابات بالضبط. تدعم دراسة التقارب أيضًا صحة حلولنا. إن نتائج بحثنا لا تساعد فقط في فهم أحداث انتشار التفاعل. كما أنه يوفر أساسًا حسابيًا مفيدًا للبحث في الأنظمة الديناميكية المتطورة.