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Research Article

Detection of hidden components (seasonal and cyclical) for long memory time series by spectral analysis of kernel estimators .

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Abstract

The long memory or long dependency (LRD) implies that the process is made up of many temporal correlations, and that the sum of the autocorrelations is slowly decreasing. Due to the length of the time series, hidden seasonal and periodic components arise in this type of series that cannot be detected through the temporal method of analysis, but through the iterative method of analysis. so non-parametric estimators have been proposed Lomax Kernel estimator, Reciprocal inverse Gaussian Kernel estimator) And compare it with by RMAD statistic. To determine the best method, simulation was used, and the results showed that the Lomax Kernel estimator was the best for its ability to detect hidden components, and it also had the lowest value for the RMAD statistic. It was applied to real data for a time series related to respiratory diseases. Kernel estimators are considered to have good advantages in the spectral analysis of time series. We also show that these infections are greatly affected by seasonality, which recurs approximately every 8 months, and its duration is not short, as it lasts for approximately 3 months.

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Preface

The long-memory time series is represented by a long-term stochastic process that has a continuity in its observations, which makes the behavior of the parameters of the autocorrelation function of the string views does not decrease exponentially towards zero when increasing the displacements ((lag) but decreases very slowly and this process is referred to as Long memory and examples of time series with long memory are the amount of flowing water, temperature, a series of a specific disease, financial circulation, etc. There are two trends in the analysis of the first time series in which the time series is a linear structure with successive limits. One of the random independent identical errors is the distribution, and it is called the time domain analysis, and the second time series is a balanced sum of the periodic sine and cosine functions. Domain, or Spectral Analysis).

The goal we want is to know the nature of the time series, the pattern it reflects, and the type of changes it contains. Here the first two approaches emerge. He sees that the time series is the result of four components (general trend, seasonal, cyclical and episodic), and the analysis aims to isolate and measure these changes. The second approach considers the series as a result of hidden sine waves of different lengths and frequencies. This analysis aims to discover the waves that have the greatest impact on the time series and determine their lengths and repetitions. This is achieved through Spectral Analysis.
Our work will be determined by using the nonparametric method in analyzing the time series with long-memory and by the spectral analysis method of the proposed kernel estimators for the purpose of using them in the nonparametric estimation of the spectral density function. that contribute to the variance of the series.

1- Time Series definition [10]

The time series is a set of observations X_t , each observation is recorded at a specific time t. Continuous time series is obtained when observations are recorded continuously over a certain period of time (continuous time series) and discrete time series is when it is taken for specific periods or times equally spaced (discrete time series).

$\emptyset_p(B)X_t = \theta_q a_t$ ……… (1)

As a_t represents the white noise that follows a certain distribution, with mean = 0, and constant variance = σ_a^2

Strings are also classified into short-memory strings and long-memory strings, or what is called Long Range Dependence (LRD) in relation to the long-term dependence of their data on some. The series with short memory is characterized by being stable in the case of taking the first or second difference with integers, . As for the long-term series, it is stable when fractional differences are taken between [-0.5, 0.5], and if it is $0 < d < 0.5$, the process is stationary with long memory.

The[15] time series X_t is defined as a series of values that are completely related to a particular phenomenon. It can be written in the form of a difference equation that represents the current values, past values, current errors, and past errors in an independent series that can be written in the following form:

$\emptyset(B)$ = [1-$\emptyset_1 B$- $\emptyset_2 B^2$ - …- $\emptyset_p B^p$] ; $\emptyset_0 = 1$ ……… (2)

$X_t = \emptyset(B)\ a_t = \sum_{i=0}^{\infty} \emptyset_i\, a_{t-i}$ ……… (3)

2- The Long Memory [13],[18]

There are several definitions of the long-term memory or long-term dependency (LRD) , They are described in the domain of time In the field of frequency Most of these definitions focus on the self-joint covariance function and the spectral density function. In order to give a description of the long memory, it is recommended to describe the stable series within the limits of the spectrum density function.

let it be[8] X_t Stable stationary in the repetition field with a function of spectral density $f(\lambda)$ The long memory appears if it is $f(0) = \infty$ so $f(\lambda)$ It has a fixed point when the frequency is equal to zero, whereas if it is $f(0) = 0$ they say that X_t Medium memory, X_t is a short memory process or short term dependence when $0 < f(0) < \infty$ because:

$$f(0) = \frac{1}{2\pi}\sum_{j=-\infty}^{\infty}\gamma(j) = 0$$

Thus, we see that the memory of the time series is of importance to measure the dependence between all the variables in the time series, as well as the effect of collecting the correlation at the same time .

In the recurrence field, I knew that the stable process Xt has a long memory with a function of spectral density and is written in the formula:

$f(\lambda) \sim C_1|\lambda|^{-\alpha}$; $0 < \alpha < 1$; $C_1 > 0$

And be [9] in a short memory in case $\alpha = 0$

And an average memory in case $\alpha < 0$; In the time domain, a stationary process is considered to have a long memory if:

$\sum_{j=-\infty}^{\infty}|\gamma(j)| = \infty$

$cov(x_t, x_{t+j}) = \gamma_j$ Where it represents the variance and the self-joint variance of the series in addition to that the self-variance decreases very slowly and follows the hyperbola.

as defined (Baillie, 1996) [12] The long memory in univariate time series, assuming that X_t is a discontinuous time process with correlation coefficient ρ_j over gaps j, so it can be said that the process has long memory if:

$\lim_{n\to\infty}\sum_{j=-n}^{n}|\rho_i| = \infty$

Where the absolute values of the correlations are non-summable , Indicates that scalar autocorrelations cannot be aggregated. It can be said that the existence of long memory processes implies that the process is composed of many temporal correlations.

The long memory streak [15] is characterized by the slow decay of autocorrelations, which makes modeling and estimation of the streak difficult. Therefore, a Fractional integration of the order d is the most used model for obtaining stability and invariance and the "long memory" came into the time series, usually combined with fractional integration.

Also [9] long memory in the Frequency Domain regresses the trend of long memory in the time domain and is known when we estimate the Spectral density function at repetitions close to zero.

3- Autoregressive Fractional Integrated Moving Average Models (ARFIMA) [5]

The ARFIMA model was introduced by Granger, Joyeux (1980) and Hosking (1981). This model proved to be more successful than the Autoregressive Integrated Moving Average (ARIMA) model. It was found that the ARFIMA model is superior to the Autoregressive Moving Average (ARMA) model and is significantly more successful in predicting time series data. ARFIMA operations (p,d,q) are widely used in LRD time series modeling, where p is the order of autoregressive , q is the order of the moving average, and d is the value of the fractional difference and the larger the value of d, the closer it is to a simple integral series ARMA]. ARFIMA models allow (d) to take any fractional value between (0) or (1) The fractionally[4] integrated series were developed according to the (ARFIMA) model according to the formula:

$$\phi(B)(1-B)^d x_t = \theta(B) a_t \quad .....(4)$$

(d): A parameter that represents a real number

α_t: It is normally distributed with a mean (zero) and a constant variance σ_α^2 .

$\theta(B), \Phi(B)$:They represent the AR and MA vehicles with recoil (B) respectively.

The Xt process is known as a process ARFIMA(p,d,q) because $\theta(B), \Phi(B)$ Polynomials for (Autoregressive) and (Moving Average), respectively, with roots outside the unit circle.

$(1 - B)^d$: The regression factor for rational differences is defined as the binomial expansion of the integer d .

$$(1-B)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)\, B^k}{\Gamma(k+1)\,\Gamma(-d)}$$

$$= 1 - dB + \frac{d(d-1)}{2!} B^2 - \frac{d(d-1)(d-2)}{3!} B^3 + \ldots$$

…..(5)

$\Gamma(.)$ Gama function from equation (5) the coefficient of the regression factor provides a decreasing ratio, the ARFIMA process is said to be a stationary process when (-0.5 < d < 0.5) However, we can divide this period into two parts [11]

1- (0 < d < 0.5) It is called a stationary long memory process.

2- (- 0.5 < d < 0) An intermediate (non-permanent) memory process is called.

And when $d \geq 1$ the process is (non-Inevitability). That is, if $|\theta| \geq 1$ then $\hat{X}_t$ (which depends on the previous values of X῾) will be infinite because the weights of $\hat{X}_{t-1}, \hat{X}_{t-2}, \hat{X}_{t-3} \ldots$ are increasing as The slowness increased and in order to avoid this situation, we put the condition |θ|≥1 to make the series $(1 - \theta B)^{-1}$ converge. We describe the series as being reversible and independent of the stability property.

4- Spectral Analysis [17],[6]

Spectral is a technique for analyzing variance (σ_x^2) across different frequencies and is one of the most widely used methods for analyzing data used with time series (Don Percival). It is also known that the studied phenomenon was described using changes of periodic time series. It is concerned with phenomena that include periodic changes that are repeated in a specific time period. The main objective of spectral analysis is to draw attention to periodic processes (Grzesicaa, Wieceka, 2016). Spectroscopy is the calculation of waves or oscillations in a set of sequential data. Where this data can be observed as a function of one or more independent variables such as spatio-temporal coordinates.

Spectral analysis [1] is one of the methods of Frequency Domain Analysis, which refers to the method given to estimate the Spectral Density Function for stationary time series, which studies the series in the frequency or frequency range. Fourier transform.

The importance of spectral analysis [1] in the study of stable phenomena or processes to know the behavior of time series and to indicate their composition and to clarify the most important compounds that contribute to the variation of the series through the contribution of frequencies of different lengths to the variance. It describes the frequency power distribution and provides information on the structure of the stochastic process, in addition to the fact that the spectrum functions have a role in the theory of linear predictions, and spectral analysis does not require specific assumptions on the structure of the process.

Spectral analysis [16] can be used to analyze the time series, explain its structure, and clarify the most important compounds that contribute to the variation of the series through the contribution of frequencies of different lengths to the variation. It also helps to know the behavior of the series itself by estimating the spectral function.

If Xt is a stable process with an absolute sum in the eigen variance series, i.e., $[\sum \{\gamma_k\}_{k=-\infty}^{\infty} < \infty]$ Using the Fourier transform of the eigenvectors, we get the following formula, which is called the power of the spectrum.

$$P(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \gamma_k\, e^{-i\omega k} \qquad -\pi \leq \omega \leq \pi$$

… (٦)

In other words:

$$P(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \gamma_k \, (\cos \omega_k \; - i \sin \omega_k) \quad -\pi \le \omega_k \le \pi$$

… (٧)

For Real Valued for the string, the above formula is written as follows:

$$P(\omega) = \frac{1}{2\pi} \{\gamma_0 + 2\sum_{k=1}^{\infty} \gamma_k \cos(\omega_k)\} \quad -\pi \le \omega \le \pi$$

… (٨)

5- The Spectral Density Function [3],[11]

The spectrum power function is the Fourier Transform of the covariance function γ_k , and it is calculated from the spectrum function shown in the following formula:

$$P(\omega) = \frac{1}{2\pi} \gamma(e^{-i\omega}) \quad -\pi \le \omega \le \pi$$

… (٩)

As long as the spectrum power P(w) does not achieve the (p.d.f) Probability Density Function which is :

1. $P(\omega) \ge 0 \quad ; \quad \forall \omega$
2. $\int_{\forall \omega} P(\omega) \; d\omega = 1$

6- Lomax Kernel function [2]

Lomax distribution: The Lomax distribution, which is also called the Pareto distribution, is a special case of the second type Pareto distribution when (m = 0) The Lomax distribution has been used in many studies in a number of fields, for example it has been widely used to model Failure times and life test as used in studies related to economics, and it was used as a proxy for the exponential distribution when the data is skewed to the right (positive skew).

The Lomax distribution has applications in the field of economic theory as well as in problems of queuing theory and in the analysis of data related to vital statistics, and

The nonparametric estimation procedure consists in estimating the parameter d first for the long memory process. We will assume that the spectral density function f is as follows:

$f(\lambda) = |\lambda|^{-2d_0} L(\lambda)$

$0 < L(\lambda) < \infty$: change slowly so that $\lambda \to 0+$, $d_0 \in (-0.5\,, 0.5)$ و $L(\lambda)$.

It is noted that the first condition is fulfilled within the period $[-\pi\,,\pi]$ or period $[0\,,\pi]$ Because of its dependence on the series of self-common variances for the continuous process in it. While the second condition is not fulfilled because:

$$\int_{-\pi}^{\pi} p(w)dw \neq 1$$

The function f(w) is called (Spectral Density Function) and is known as :

$$f(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \frac{\gamma_k}{\gamma_0} e^{-i\omega k}$$

$$= \frac{1}{2\pi}[1 + \frac{2}{\gamma_0} \sum_{k=1}^{\infty} \gamma_k \cos(\omega_k)] \quad -\pi \le \omega \le \pi$$

… (١٠)

the use of Bayesian estimation to find Bayesian limits for prediction. Bayes estimators, and the confidence interval of the reliability function when the measurement parameter is constant.

Density distribution function: X is said to be a random variable that follows a (Lomax) distribution with the two parameters α , β . denoted by $X \sim L\,(\alpha, \beta)$ If the probability density function is (pdf) Knowledge of the general formula for the Lomax distribution .

$f(x, \alpha, \beta) = \frac{\alpha}{\beta}\left(1 + \frac{x}{\beta}\right)^{-(\alpha+1)} \qquad x > 0\;, \alpha, \beta > 0$

(α) : represents the shape parameter, β : represents the scaling parameter and when the scaling parameter value is equal to one we get the probability density function of the Lomax distribution with respect to the shape parameter (α)

$$\boldsymbol{f(x,\alpha) = \alpha(1+x)^{-(\alpha+1)} \qquad x>0\,,\alpha>0}$$

The Lomax distribution density function is given by the following formula:

$$\mu(t) = \frac{\alpha}{\lambda}\left(1+\frac{t}{\lambda}\right)^{-(\alpha+1)} \quad \dots (١١)$$

λ > 0 the scaling parameter ،α > 0 the shape parameter . The mean of this distribution is: $Mean = \frac{\lambda}{\alpha-1}$, $\alpha > 1$ And the variance: $Var = \frac{\lambda\alpha}{(\alpha-1)^2(\alpha-2)}$, $\alpha > 2$

The cumulative distribution function of the Lomax distribution is as follows:

$$F(t) = 1-\alpha(1+t)^{-(\alpha+1)} \quad \dots (١٢)$$

Similarly, in the case of λ = 1, the Lomax distribution shifts to the exponential distribution with the parameter α. The first moment [7] of the Lomax distribution is present and equal to $\frac{1}{\alpha-1}$ only if α > 1 . Suppose we have an operation $(X_1, X_2, \dots, X_T)$ From a stable process that has a spectral density function as follows:

$$f(\lambda) = \sum_k \gamma(k)\exp(-2\pi\lambda k) \quad \dots (١٣)$$

7- Reciprocal inverse Gaussian Kernel[14]

The name "inverse Gaussian" was introduced by Tweedie (1947) who observed the inverse relationship between the cumulative generating functions of these distributions and those of Gaussian distributions. They are also known as "Wald" distributions because the same class of distributions was derived by Wald (1947). The RIG kernel has a flexible shape and location on the non-negative real line. Their shapes are allowed to vary according to the position of the data points, thus varying the degree of smoothing in a natural way, and their support corresponds to that of the probability density function under estimate.

The RIG estimator is borderline-bias-free, always non-negative, and achieves an optimal mean-squared error (MISE) convergence

$\gamma(k)$ The covariance function of the process X_t . For simplicity, we assume that a stationary process has a mean of zero. The Periodogram scheme is as follows:

$$I_T(w_j) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} X_t \exp(-2\pi i w_j t)\right|^2 \quad , \quad w_j = \frac{i}{T}, j = 1, \dots\dots, T/2 . \quad \dots (١٤)$$

It is known to be an asymptotic unbiased estimator of the spectral density function f .

A consistent estimator was found after appropriate smoothing of the I_T over a range of frequencies. The following estimate will be studied:

$$\hat{f}(\lambda) = \frac{1}{T}\sum_{j=1}^{T} K_{b,\lambda}(w_j) I_T(w_j) \quad \dots (١٥)$$

$K_{b,\lambda}$: It is an Lomax kernel function, which is defined as:

$$K_{\alpha,\lambda} = \frac{\left(\frac{1-\lambda}{b}\right)}{\left(1+\frac{1-\lambda}{b}\right)}\left(1+\frac{w}{1+\frac{1-\lambda}{b}}\right)^{-\frac{\lambda}{b}} I_{0\le w\le 1} \quad \dots (١٦)$$

For the Lomax function and the smoothing parameter b. The Lomax cortical function is the probability density function of a random variable that has a Lomax distribution with the parameters

$$\left(1+\frac{\lambda}{b}, 1+\frac{1-\lambda}{b}\right)$$

within the category of non-negative kernel density estimators. Moreover, its contrast decreases as the position at which smoothing is done moves away from the boundary. In contrast to the gamma kernel estimators, the RIG kernel estimator avoids having the first derivative of the probability density function in its bias.

Let $(X_1, X_2, \dots, X_n)$ A random sample from the distribution of an unknown probability density function f defined over the interval [0, ∞) and that f is differentiated twice continuously and $\int_0^\infty (x^3 f''(x))^2 dx < \infty$

Let $K_{IG(m,\lambda)}$ It is the probability density function of the Gaussian distribution with the random variable Y defined as:

$$K_{IG(m,\lambda)} = \frac{\sqrt{\lambda}}{\sqrt{2\pi y^3}}\exp\left(-\frac{\lambda}{2m}\left(mz-2+\frac{m}{y}\right)\right) \;; y>0 \quad \dots (١٧)$$

The mean and variance of Y are: $E(Y) = m$, $Var(Y) = \frac{m^3}{\lambda}$

random variable $z = \frac{1}{Y}$ Behaves $RIG(m, \lambda)$ Which has a probability density function as follows:

$$K_{RIG(m,\lambda)}(z) = \frac{\sqrt{\lambda}}{\sqrt{2\pi z}} \exp\left(-\frac{\lambda}{2m}\left(mz - 2 + \frac{1}{mz}\right)\right) \; ; z > 0 \quad \ldots (١٨)$$

The mean and variance of z are: $E(Z) = \frac{1}{m} + \frac{1}{\lambda}$, $Var(Y) = \frac{1}{\lambda m} + \frac{2}{\lambda^2}$

The RIG class of kernel functions takes into account:

$$K_{IG\left(x,\frac{1}{b}\right)}(u) = \frac{1}{\sqrt{2\pi b u^3}} \exp\left(-\frac{1}{2bx}\left(\frac{u}{x} - 2 + \frac{x}{u}\right)\right) \; ; \quad \ldots (١٩)$$

$$K_{RIG\left(\frac{1}{x-b}, \frac{1}{b}\right)}(u) = \frac{1}{\sqrt{2\pi b u}} \exp\left(-\frac{x-b}{2b}\left(\frac{u}{x-b} - 2 + \frac{x-b}{y}\right)\right) \; ; \quad \ldots (٢٠)$$

Since b is the smoothing parameter, which is achieved $b + 1/(bn) \to 0$ when n go to infinity. The probability density functions are:

$$\hat{f}_{IG}(x) = n^{-1} \sum_{i=1}^{n} K_{IG\left(x,\frac{1}{b}\right)}(X_i) \quad \ldots (٢١)$$

$$\hat{f}_{RIG}(x) = n^{-1} \sum_{i=1}^{n} K_{RIG\left(\frac{1}{x-b}, \frac{1}{b}\right)}(X_i) \quad \ldots (٢٢)$$

The estimators in equations (21) and (22) are very easy to implement, and are very similar to the gamma kernel estimators. It is obtained after replacing the RIG kernel with the gamma kernel used by Chen (2000)), and it is either:

$$K_{Gam(x/b+1,b)}(u) = \frac{u^{x/b} e^{-u/b}}{b^{\frac{x}{b}+1}\Gamma(x/b+1)} \quad ; u > 0 \quad \ldots (٢٣)$$

Or :

$$K_{Gam(\rho b(x),b)}(u) = \frac{u^{\rho b(x)-1/b} e^{-u/b}}{b^{\rho b(x)}\Gamma(\rho b(x))} \quad ; \; u > 0 \quad \ldots (٢٤)$$

$$\rho b(x) = \begin{cases} \frac{x}{b} & \text{if } x \geq 2b \\ \frac{1}{4}(x/b)^2 + 1 & \text{if } x \in [0, 2b) \end{cases} \quad \ldots (٢٥)$$

Thus, the kernel estimator for the distribution is as follows:

$$K_{RIG} = \frac{\sqrt{\left(\frac{1-\lambda}{b}+1\right)}}{\sqrt{2\pi\omega}} e^{\frac{\left(\frac{1-\lambda}{b}+1\right)}{2\left(\frac{\lambda}{b}+1\right)}\left(\left(\frac{\lambda}{b}+1\right)\omega - 2 + \frac{1}{\left(\frac{1-\lambda}{b}+1\right)\omega}\right)} \quad \ldots\ldots(26)$$

When estimating the value of λ, we get the different frequencies that enable us to detect periodic and seasonal compounds in the time series.

8- simulation

Several values were used for the Hurst coefficient (H), which determines the value of the fractional difference (d) and these values are (H=0.1, H=0.4, H=0.6, H=0.9), which results in d values through the relationship that links Hearst's coefficient H with the degree of fractional integration d:

$d = H - 0.5$

also adopted a set of sample sizes (N=500, N=750, N=1000) for the purpose of knowing the behavior of the estimators used. In addition, the spectrum function estimator was calculated according to the previously mentioned methods with the proposed methods, and their estimations were drawn as in Figure (1,2). The simulation experiment was repeated 1000 times for the purpose of arriving at stable and reliable estimates in the comparison process, and the results were as in the following tables:

Table (1) shows the values of RMAD and evaluate S.D When H=0.1

N =500			N =750			N =1000		
Lomax Kernel			Lomax Kernel			Lomax Kernel		
b	RMAD	s.d	b	RMAD	s.d	b	RMAD	s.d
0.005	0.61713	0.06959	0.005	0.62324	0.05918	0.005	0.62488	0.05767
0.01	0.61793	0.06905	0.01	0.62355	0.05885	0.01	0.62526	0.05742
0.05	0.61836	0.07213	0.05	0.62245	0.06173	0.05	0.62395	0.06028
0.08	0.61679	0.0756	0.08	0.62066	0.06441	0.08	0.62211	0.06338
inverse Gaussian Kernel			inverse Gaussian Kernel			inverse Gaussian Kernel		
b	RMAD	s.d	b	RMAD	s.d	b	RMAD	s.d
0.005	0.67625	0.16399	0.005	0.66114	0.10931	0.005	0.65723	0.08666
0.01	0.65833	0.12143	0.01	0.64932	0.09489	0.01	0.64798	0.07918
0.05	0.64273	0.10154	0.05	0.63885	0.08196	0.05	0.63925	0.07556
0.08	0.64064	0.09914	0.08	0.6377	0.08073	0.08	0.63816	0.0757

Table (1) Monte Carlo simulation results for FARIMA(1,H=0.1,0) with $\alpha = 0.6$ (RMAD) Relative Mean Absolute Deviation.

It is clear from the table that the Lomax Kernel estimator had the lowest values in the RMAD criterion, regardless of the difference in sample size, as we observe at b = 0.08 and N = 500, which gave the best results.

Table (2) shows the values of RMAD and evaluate S.D When H=0.4

N =500			N =750			N =1000		
Lomax Kernel			Lomax Kernel			Lomax Kernel		
b	RMAD	s.d	b	RMAD	s.d	b	RMAD	s.d
0.005	0.61299	0.06632	0.005	0.62082	0.0629	0.005	0.6256	0.05878
0.01	0.614	0.06542	0.01	0.62177	0.06245	0.01	0.62613	0.05852
0.05	0.61503	0.06824	0.05	0.62188	0.06463	0.05	0.62527	0.06086
0.08	41٢0.61	0.072	0.08	0.62001	0.06717	0.08	0.62336	0.06311
inverse Gaussian Kernel			inverse Gaussian Kernel			inverse Gaussian Kernel		
b	RMAD	s.d	b	RMAD	s.d	b	RMAD	s.d
0.005	0.67366	0.17742	0.005	0.65476	0.09547	0.005	0.64708	0.08335
0.01	0.65509	0.12854	0.01	0.64511	0.08442	0.01	0.64043	0.07637
0.05	0.63732	0.09271	0.05	0.63499	0.07611	0.05	0.63297	0.07052
0.08	0.63507	0.08916	0.08	0.63361	0.07512	0.08	0.63203	0.07009

Table (2) The same process except that H = 0.4, that is, the time series is still dependent on a long range, Monte Carlo simulation results for FARIMA(1,H=0.4,0) with $\alpha = 0.6$ with (RMAD) Relative Mean Absolute Deviation . It is clear from the above table that the Lomax Kernel estimator had the lowest values in the RMAD criterion, regardless of the difference in the sample size, as we note at b = 0.08 and N = 750, which gave the best results.

Table (3) shows the values of RMAD and evaluate S.D When H=0.6

N =500			N =750			N =1000		
Lomax Kernel			Lomax Kernel			Lomax Kernel		
b	RMAD	s.d	b	RMAD	s.d	b	RMAD	s.d
0.005	0.61297	0.06401	0.005	0.62553	0.06432	0.005	0.62189	0.05428
0.01	0.61321	0.06356	0.01	0.62567	0.06415	0.01	0.62216	0.05404
0.05	0.61188	0.06761	0.05	0.62411	0.06763	0.05	0.62093	0.05737
0.08	0.61092	0.07069	0.08	0.62319	0.07126	0.08	0.61979	0.0608
inverse Gaussian Kernel			inverse Gaussian Kernel			inverse Gaussian Kernel		
b	RMAD	s.d	b	RMAD	s.d	b	RMAD	s.d
0.005	0.66729	0.14785	0.005	0.65671	0.11218	0.005	0.6467	0.07561
0.01	0.65016	0.09648	0.01	0.64842	0.08871	0.01	0.64144	0.07068
0.05	0.63649	0.07593	0.05	0.64008	0.07462	0.05	0.63578	0.06621
0.08	0.635	0.07512	0.08	0.63888	0.07378	0.08	0.63484	0.06562

Table (3) Results of Monte Carlo simulation for FARIMA(1,H=0.6,0) model with $\alpha = 0.6$ with Relative Mean Absolute Deviation (RMAD). It is clear from the table that the Lomax Kernel estimator had the lowest values in the RMAD criterion, regardless of the difference in the sample size, as we notice at b = 0.08 and N = 500, which gave the best results.

Table (4) shows the values of RMAD and evaluate S.D When H=0.9

N =500			N =750			N =1000		
Lomax Kernel			Lomax Kernel			Lomax Kernel		
b	RMAD	s.d	b	RMAD	s.d	b	RMAD	s.d
0.005	0.63702	0.06999	0.005	0.64111	0.05912	0.005	0.64692	0.05619
0.01	0.63156	0.06971	0.01	0.63415	0.05825	0.01	0.63903	0.05554
0.05	0.6203	0.0811	0.05	0.6227	0.06897	0.05	0.6296	0.06867
0.08	0.62816	0.09056	0.08	0.63232	0.07879	0.08	0.6433	0.07946
inverse Gaussian Kernel			inverse Gaussian Kernel			inverse Gaussian Kernel		
b	RMAD	s.d	b	RMAD	s.d	b	RMAD	s.d
0.005	0.69784	0.10744	0.005	0.68277	0.08657	0.005	0.67303	0.07908
0.01	0.69344	0.10426	0.01	0.67847	0.08572	0.01	0.67064	0.07873
0.05	0.68608	0.10085	0.05	0.67132	0.08413	0.05	0.66596	0.0773
0.08	0.68412	0.10019	0.08	0.66948	0.08367	0.08	0.66436	0.07665

Table (4) Monte Carlo simulation results for FARIMA(1,H=0.9,0) model with $\alpha = 0.6$ with Relative Mean Absolute Deviation (RMAD). It is clear from the table that the Lomax Kernel estimator had the lowest values in the RMAD criterion, regardless of the sample size difference, as we note at b = 0.05 and N = 500, which gave the best results.

3- Plotting time series and correlation functions

For the purpose of clarifying part of the simulation method and the data of the time series generated according to the models that the researcher relied on in the simulation process, a model of the time series was drawn, as in the following figure that follows the ARFIMA model(1,H=0.6,0), where α = 0.6 and the value of H=0.6 .

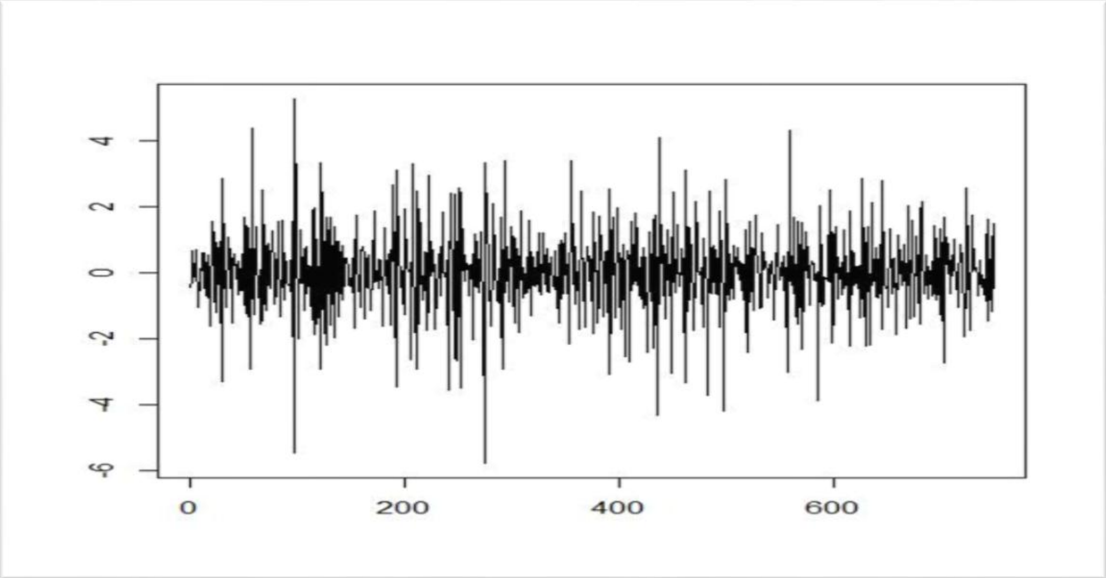

Figure (1) A time series generated according to a model ARFIMA (1,H=0.6,0)

The ACF autocorrelation functions were also found and plotted as in the figure below, and the behavior of the generated model is clear according to ARFIMA(1,H=0.6,0)

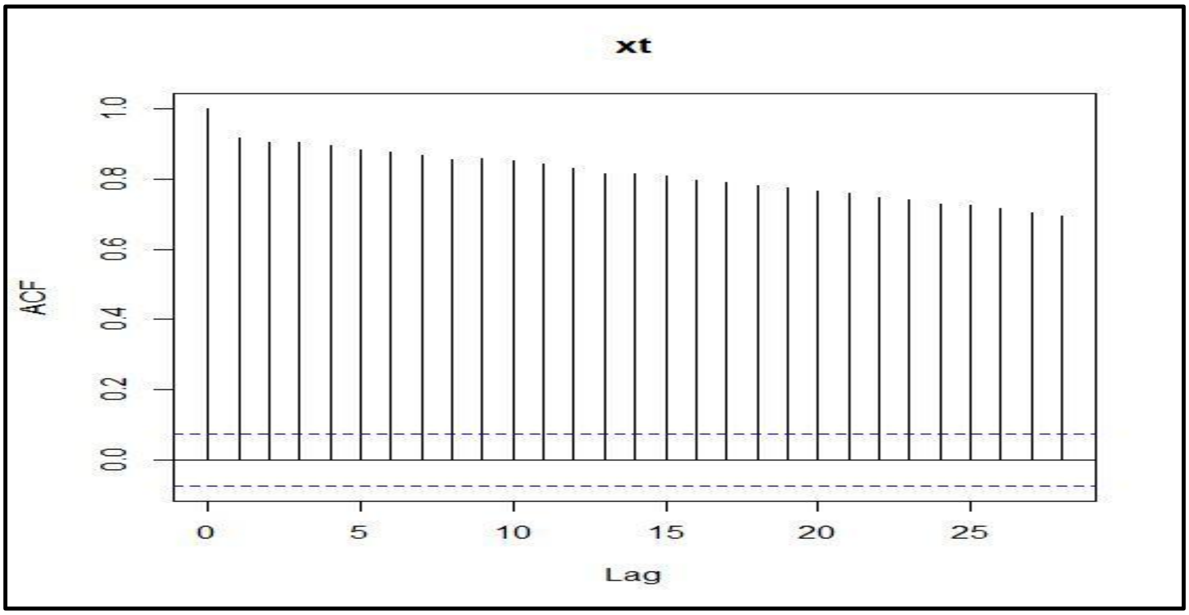

Figure (2) The experimental autocorrelation function ACF

The ACF diagram of the data generated for the ARFIMA(1,H,0) model shows that the time series is characterized by the long memory feature, and this is evident by the very slow decreasing of the ACF function over the long time gaps.

4- spectrum function diagram

A- Spectrum function using Lomax kernel estimator

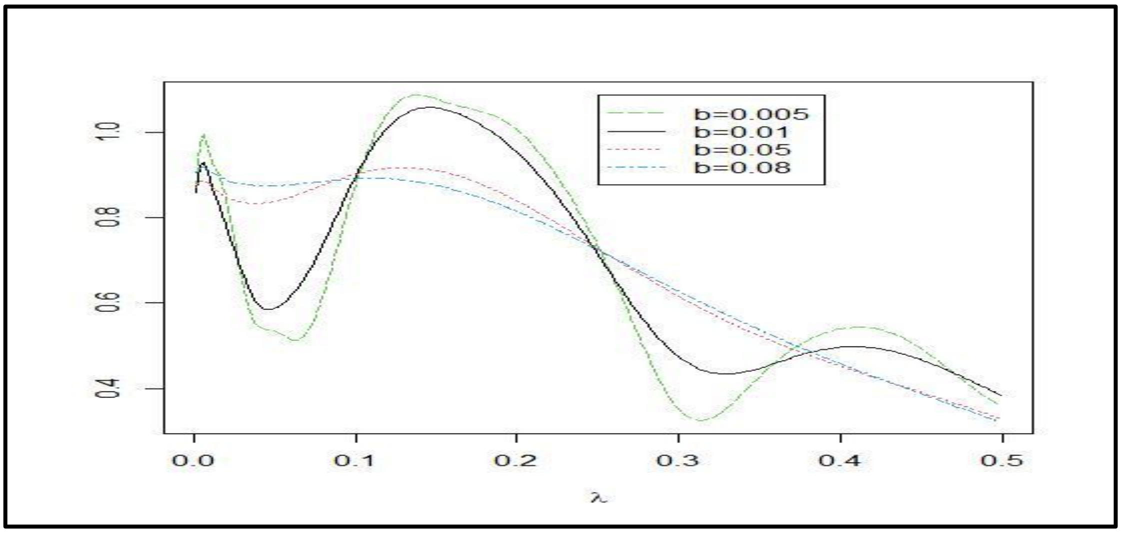

Figure (3) Logarithm values of the spectrum estimated using the Lomax kernel estimator logarithm of the logarithm series of data with different bandwidth values

As for the spectrum function of the logarithm series of the data estimated using the Lomax Kernel estimator, as we notice from Figure (3) that the spectrum frequency varies according to the different values of the bandwidth, as the spectrum function was high at the bandwidth b = 0.005 and at the point of origin, i.e. the frequency value λ = 0.0, then it decreased rapidly Sharp at frequency λ = 0.07, then re-rise at λ = 0.2, then continued to decrease at frequency λ = 0.3, and returned to rise at frequency λ = 0.4, but at lower levels, then to reach the lowest level at frequency λ = 0.5, as well as at bandwidth 0.01 behaved the same behavior but at a level Less high and low, while the rest of the values of the band b = (0.05, 0.08) started to decrease at λ = 0.05 and rise at λ = 0.15 at a simple pace at high frequencies.

B- Spectrum function using Reciprocal inverse Gaussian Kernel

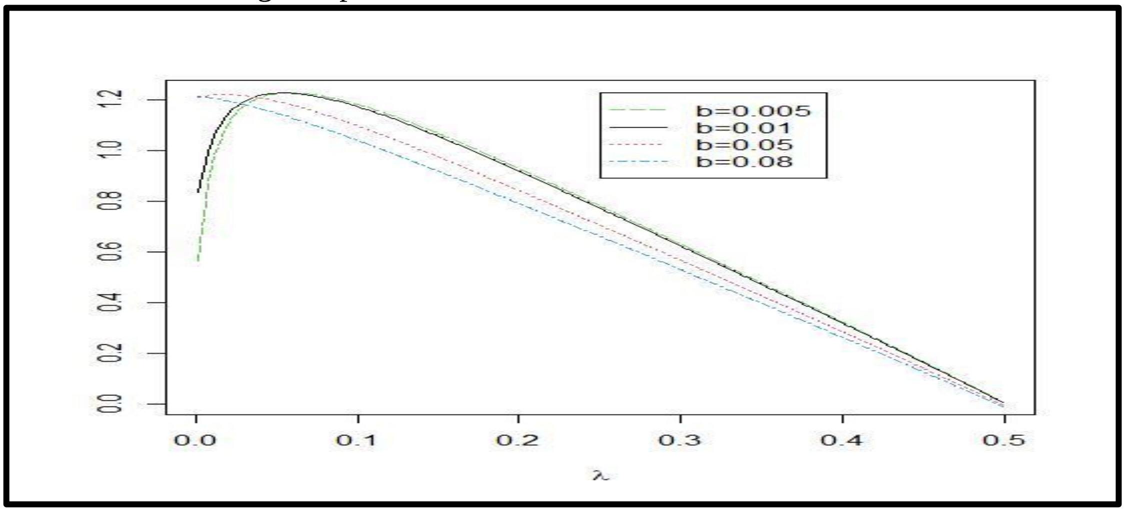

Figure (4) Log spectrum logarithm values of the logarithm series of data estimated using a Reciprocal inverse Gaussian Kernel estimator with different bandwidth values.

We notice from Figure (4) that the spectrum frequency at λ = 0 is the lowest at the bandwidth b = 0.005, then at b = 0.01 at a higher level, but at the values of b = (0.05, 0.08) the spectrum function is high at the frequency λ = 0 Then it starts decreasing gradually to reach the lowest level at λ = 0.5 for all bandwidth.

9- real data

(data) (respiratory disease incidence)

Data representing a time series for the number of cases of respiratory diseases per day for the period (2018-2019) were taken from the Health and Life Statistics Division in the Babel Health Department through the monthly schedule of the auditors of the consulting clinics in hospitals in Babylon Governorate.The following table shows the descriptive statistics of the data and the logarithm of the data for the time series

Table (5) of descriptive statistics

Max	Min	C.V	Variance	Std.Devation	Mean	
600	200	30.404	14736.302	121.393	399.26	Data
2.778	2.301	5.488	0.020	0.141567	2.5793	log Data

he results in Table (5) show that the arithmetic mean value of the number of infected people was 399.26, while the mean of the logarithm data was 2.579. The table also shows the value of the standard deviation, variance and coefficient of variation. In addition, the time series for the original data was plotted as follows:

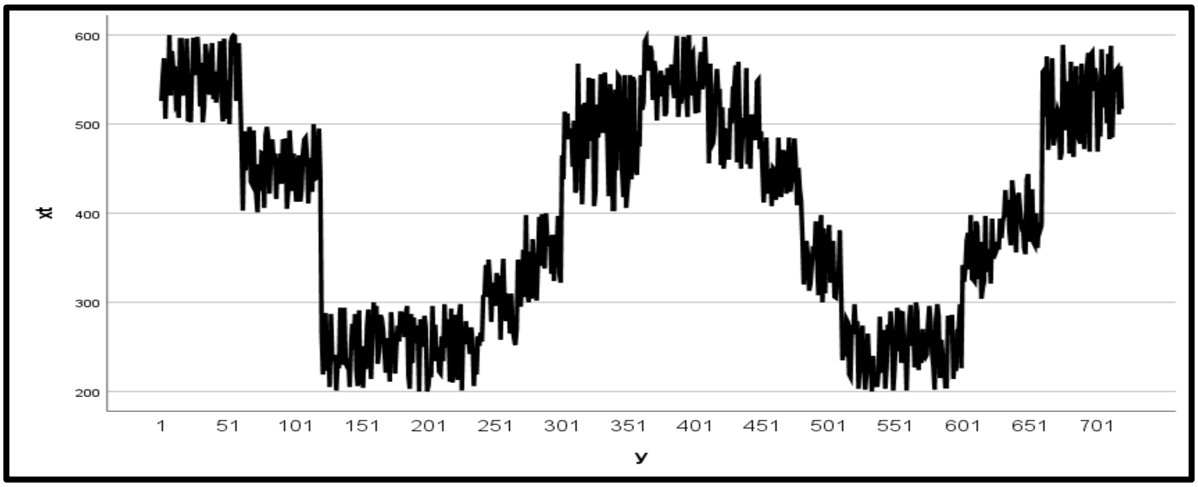

Figure (5) plotting the time series of the original data

By drawing the time series, we notice that it is stable in the mean (399.26), but it is unstable in the variance, and this is evident through the fluctuation around the mean, so the natural logarithm was taken to get rid of the variance in the data, then the autocorrelation function was drawn and the series was plotted for the logarithm of the data, as follows:

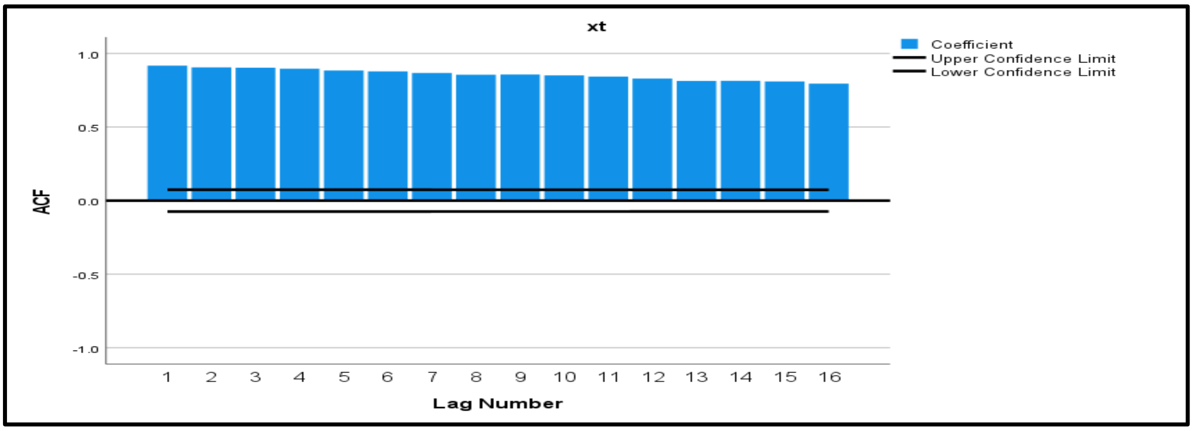

Figure (6) Autocorrelation function of the original data of the time series

By drawing the ACF autocorrelation function, it is clear that the time series is characterized by the long memory feature, and this was inferred by the slow gradual decrease in the autocorrelation values

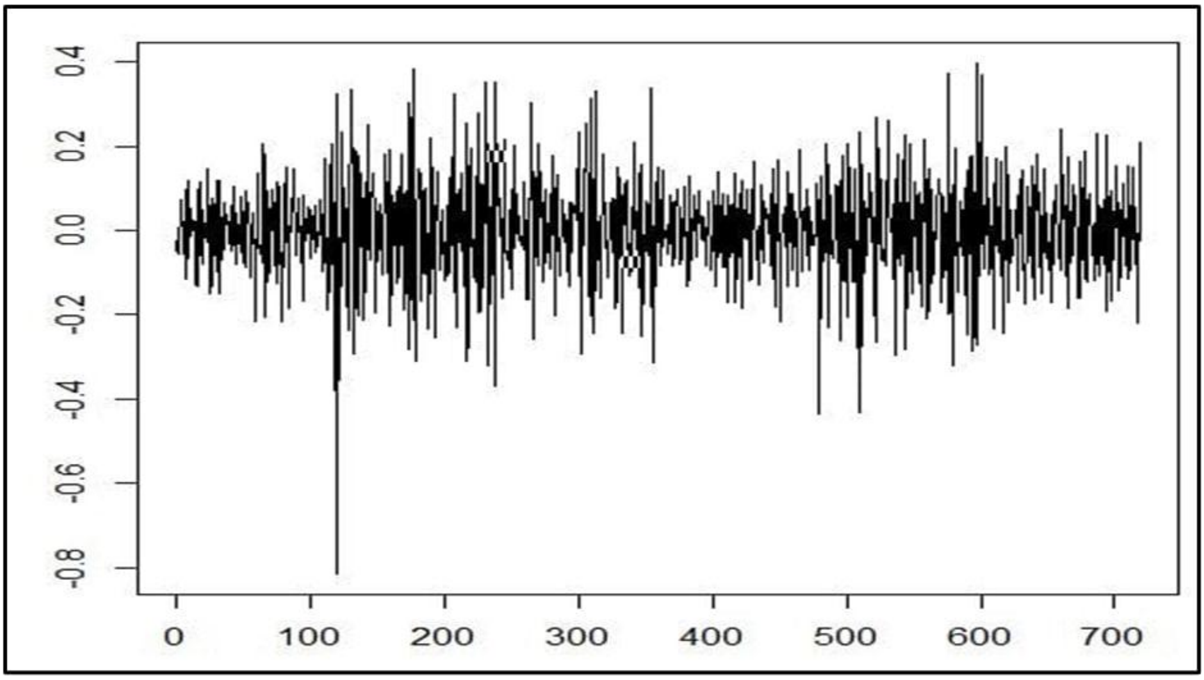

Figure (7) Time series and logarithm of the data

Through Figure (7) we note the stability of the time series in the arithmetic mean equal to 2.5793, but we note that there is fluctuation around the arithmetic mean in different periods of the time series. For the purpose of knowing the inference about the long memory of the time series above, the autocorrelation function (ACF) was drawn, as shown in the following figure:

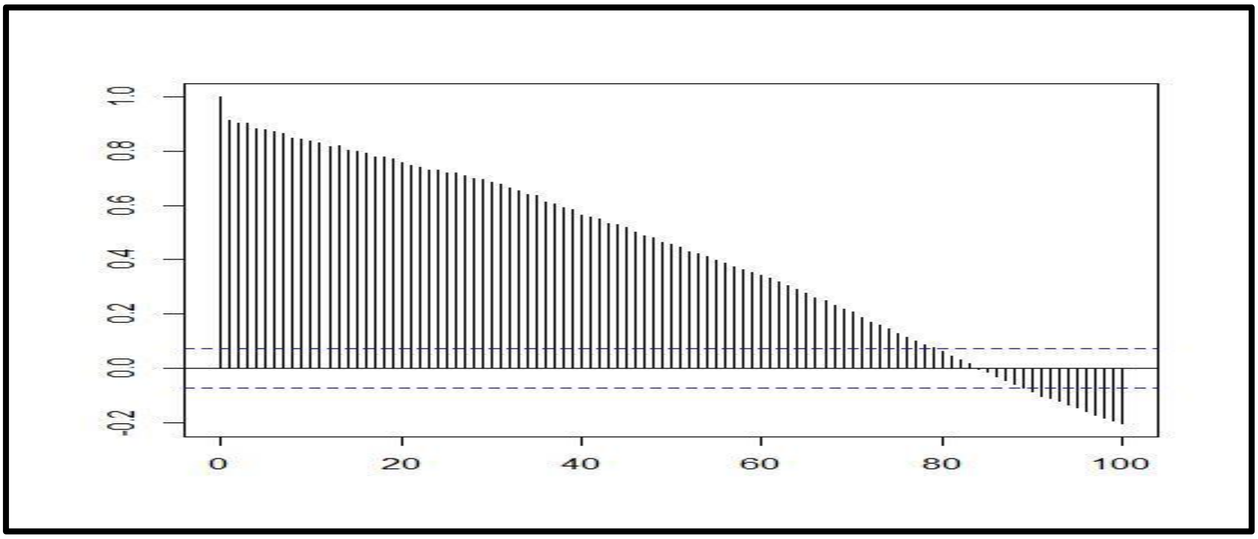

Figure (8) the autocorrelation function of the logarithm of the data for the time series

The drawing of the ACF autocorrelation function as shown in Figure (8) shows that the time series is characterized by the long memory feature, and this is evident by the very slow decrease of the ACF function over the long-time gaps.

Long memory tests

H_0 : There is a long memory in the time series .

H_1 : There is a short memory in the time series .

To achieve this, five long memory tests were used, which are the modified Hurst statistic: The modified Hurst statistic, KPSS test, V/S statistic, The Kolmogorov statistic, Anderson–Darling statistic, and Table (4-3) representing the value of a statistic (H-value) for each test:

Table (6) Results of long series memory tests

Statistics test	H. value
Simple R/S Hurst estimation	0.5744465
Corrected R over S Hurst exponent	0.5947502
Empirical Hurst exponent	0.5619452
Corrected empirical Hurst exponent	0.5324144
Theoretical Hurst exponent	0.5443251

It is clear from the previous table through the value of H. value that the value of the parameter $0 < d < 0.5$ means that the data has a long memory.

Fractional parameter estimation

The parameter of fractional differences was estimated according to the method of GPH Geweke and Porter-Hudak, and the estimate value was d = 0.488. Thus, the stable long memory property is achieved according to the rule $0 < d < 0.5$.

Estimation of spectral analysis function

Estimation of the spectrum function logarithm of the absolute data based on the Lomax kernel estimator with different bandwidth values

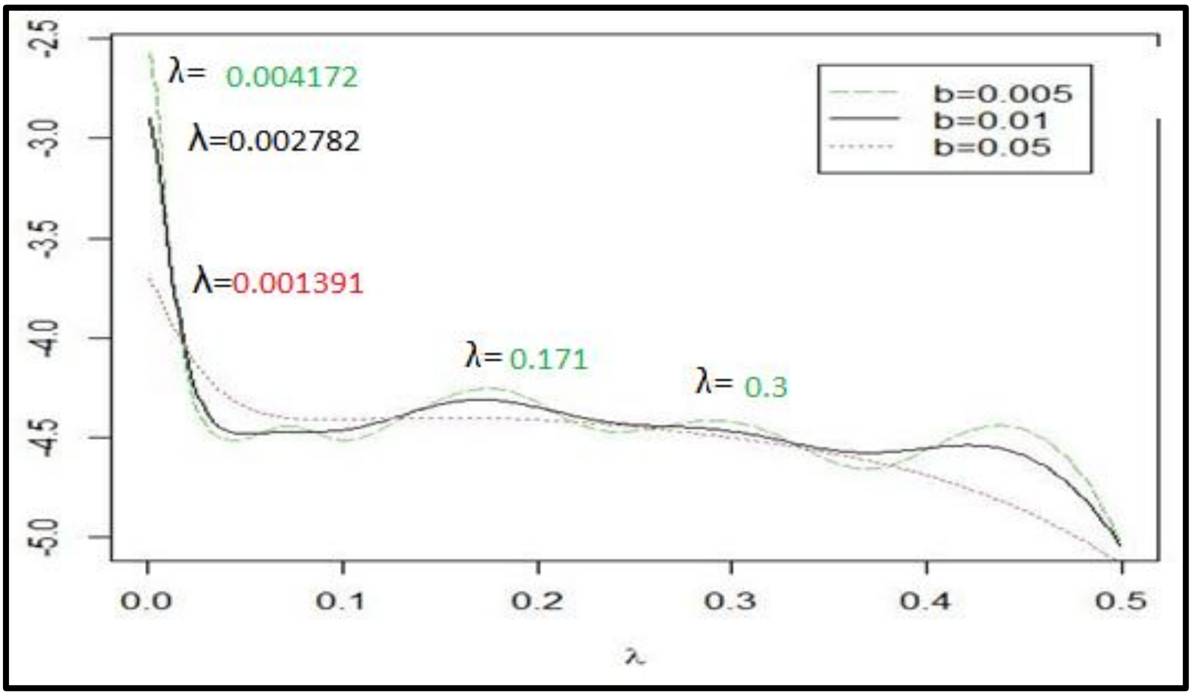

Figure (9) The logarithm of the spectrum logarithm of the logarithm series of absolute data estimated using a Lomax Kernel estimator with different beamwidth values.

From the figure, we conclude that there is a seasonal or periodic vehicle with a duration of 93 days (about 3 months), which is the difference between the peak at frequency (λ = 0.17) and the other peak at frequency (λ = 0.3). (8 months) by taking the inverse of the highest frequency at (bandwith = 0.005) and (0.004172) = λ), which is equal to $\frac{1}{0.004172} = 239.69$ day can also be calculated from the bottom, which is the lowest frequency at λ = 0.1, to the next lowest frequency at λ = 0.23 to have 94 days.

10- Conclusions

1. It was found from the RMAD values that the best estimator is the Lomax Kernel estimator and that increasing the sample size reduces the RMAD values of the estimators
2. The estimation of the spectrum function was clearly demonstrated by drawing the spectrum function to the presence of the hidden components of the long-memory time series behavior.
3. It was found through drawing the autocorrelation functions and through the long memory tests that the studied time series is characterized by the characteristic of long-term dependence (long memory). It was also found from the estimation of the fractal parameter that the time series has a long memory and is characterized by stability.
4. By comparing between the estimators, it was found that the Lomax estimator is the lowest RMAD, and thus it is considered the most accurate among the estimators, and that it is an estimator that achieves an optimal convergence rate in the mean squares of errors and approaches the real values faster than the rest of the estimations under study.
5. By estimating the spectral density, it was possible to clarify the hidden compounds that characterize the time series and know their characteristics (duration of 3 months and repeated every 8 months) by knowing the values of frequencies λ.
6. Kernel estimators are considered to have good advantages in spectral analysis of time series in case the distribution is not known or the assumptions are difficult in the case of parameter estimation.
7. We recommend using the Lomax Kernel estimator in estimating the spectrum function to distinguish it from the rest of the estimators because it showed the lowest RMAD value. Also, the Lomax estimator was able to detect hidden compounds in the time series with a long reliability of the data.
8. The mentioned non-parametric estimators can be considered as a basic rule for the purpose of estimating the required data. Also, despite our goal is long memory, the capabilities can be worked on short memory as well.
9. We recommend officials in the Ministry of Health to give great attention to this type of injuries, limit their spread and find appropriate solutions. We also show that these injuries are greatly affected by seasonality, which recurs approximately every 8 months, and its duration is not short, as it lasts for approximately 3 months.

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