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Even Sum Edge Domination in Graph

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P A P E R I N F O

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A B S T R A C T

Various applications of domination and recent work on different forms of domination in graphs have given rise to our interest in exploring in this paper new special type of graph domination. As most papers discussed domination focus more on setting the conditions for the dominant group to come up with a new concept of domination, this pape introduces the new parameter of domination in graph called even sum edge domination set (ESEDS). Many of properties of these numbers are being discussed. Furthermore, for the path and related to it as thorn, thorn path, and thorn rod path are presented. Also, the algorithm of even sum edge domination of path is introduced.

1. INTRODUCTION

The domination graph is very important number in variant fields in sciences and many applications in real life as in physics, chemistry, biology, engineering, medicine and others. This concept is first appeared in 1962 by Berge [1] and then this concept is used to find solution to many problems in various sciences. The new concept of even sum domination set on vertices was defined by Rasheed and Omran [2]. They have studied many properties of this concept on certain graphs [3]. The concept of domination graph deals with different fields in mathematics such as labeled graph [4], topological graph [5-8], fuzzy graph [10,11], and general graph [9] and [12,13]. This work deals with the undirected finite simple graph. The edge even sum domination introduces and discusses many properties of this new concept. Many theorems, properties, corollary are proved. For path and related path as thorn path and thorn rod path the even edge domination is determined. Additionally. The algorithm of even sum edge domination is calculated. More details about the concepts in graph theory can be found in [6,7].

2. BASIC DEFINITIONS

Definition .1: The edge degree $d(e)$ of the edge $e = uv$ is defined as the number of neighbors (a common vertex with the edge e) of e, i.e., $|N(u) + N(v) - 2|$. Definition .2: Let (V, E) be a graph that has no isolated vertex and F is a set of edges, the set F is called an edge dominating set (EDS) if, for each $e_i \in E - F$ there is an edge $e_j \in F$ adjacent to e_i (every edge not in F adjacent to at least one edge in F).Definition .3: Let (V,E) be a graph that has no isolated vertex and F is an EDS, the set F is called even sum edge domination set (ESEDS) if for every $e_i \in E - F$ there is an edge $e_j \in F$adjacent to e_i such that $deg(e_i) + deg(e_j)$ is even.
Definition .4: Let D be an ESEDS in a graph, if the set D has no proper ESEDS, then this set is called minimal ESEDS (MESDS). Furthermore, the even sum domination number is the minimum cardinality of all MESEDS, and denoted by $\gamma'_{es}(G)$.

If D is an MESEDS that has the minimum cardinality, then D is called γ'_{es}-set.
Observation: Assume that G is a graph with n vertices and D is a γ'_{es} −set, then
1) Each K_2 component in graph G belongs to every ESEDS.

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2) Every edge e_i in the set $V - D$ has an even (odd) degree is adjacent to at least one edge e_j in D that has an even (odd) degree.
3) An edge has an odd (even) degree that belongs to every ESEDS, if it is not adjacent to edge has odd (even) degree.
4) $\gamma'_{es}(G) \leq \gamma'(G)$.

3. THE MAIN RESULTS

Proposition .1: Let G be a regular graph, then $\gamma'_{es}(G) = \gamma'(G)$.

Proof. Let G be a regular graph, so the degree of all vertices is equal say r. Thus, the degree of all edges is $(2r-2)$.So, there are two cases: one of them $(2r-2=0)$, therefore $G \equiv K_2$ otherwise $(2r-2)$ is even.

Thus, $\gamma'_{es}(G) = \gamma'(G)$.

Remark .2: The converse of the previous proposition is not necessarily true. For example, take the star graph (S_6) where $S_6 \equiv K_{1,5}$.

Corollary 3.
1) $\gamma'_{es}(K_n) = \gamma'(K_n) = \left\lceil \frac{n}{2} \right\rceil$.
2) $\gamma'_{es}(C_n) = \gamma'(C_n) = \left\lceil \frac{n}{3} \right\rceil$.

Proposition. 2: Let G be a path of order $n; n \geq 2$, so

$$\gamma'_{es}(P_n) = \begin{Bmatrix} 1, & if\ n = 2,3 \\ 2 + \left\lceil \frac{n-3}{3} \right\rceil, & if\ n \geq 4 \end{Bmatrix}.$$

Proof. The proof is treated separately for two cases.

Case 1. If $n = 2$, then $P_2 \equiv K_2$, thus $\gamma_{es}(P_2) = 1$. Also, if $n = 3$, then there are only two disjoint edges each of them of degree 3, so $\gamma_{es}(P_3) = 1$.

Case 2. If $n \geq 4$, then the degree of each pendant edge is 1 and not adjacent to an edge of odd degree. Thus, by Observation 5, these edges belong to every ESEDS. These two edges do not dominate to adjacent edges since these have an odd degree and the adjacent have an even degree. All remained edges have even degree, so all three consecutive edges can be dominated by one edge. The three consecutive edges are covering four vertices thus, $\gamma_{es}(P_n) = 2 + \left\lceil \frac{n-3}{3} \right\rceil$.

From the cases above, the result is obtained.

Algorithm .1: ESEDS-PATH (G is a path of order n)

Input: the vertex set $V = \{v_1, v_2, \dots, v_n\}$, $v_1$and v_n are the pendant vertices

1. $ESEDS := \emptyset$;
2. Case 1: $n = 2\ or\ 3$;
3. $ESEDS := \{e_1\}$;
4. Case 2: $n \geq 4$;
5. $ESEDS := \{e_1, e_n\}$;
6. For $i :=\ 3\ \ to\ \ n-1$ step 2, do;
7. $ESEDS := ESEDS \cup \{e_i\}$;
8. end for; Output: $ESDS$ Proposition .3: Let G be a thorn path $P_{n,r,s}$. Then

$$\gamma'_{es}(P_{n,r,s}) = \begin{Bmatrix} n + \left\lceil \frac{n}{3} \right\rceil, & if\ s\ is\ odd\ and\ r\ is\ even \\ 2 + \left\lceil \frac{n-4}{2} \right\rceil, & if\ s\ \ and\ r\ are\ odd\ or\ s\ is\ even\ \ and\ r\ is\ odd \\ n + \left\lceil \frac{n-2}{3} \right\rceil, & if\ s\ and\ r\ are\ even \end{Bmatrix}$$
.

Proof. The proof is treated separately for three cases.

Case 1. If $s\ is\ odd\ and\ r\ is\ even$, then all terminal edges in the thorn graph have an odd degree and all edges of the path graph (non-terminal edges in the thorn graph) have an even degree (as an example, see Figure 1). Thus, the terminal edges do not dominate the non-terminal edges, so take one edge from each terminal that is adjacent to a vertex (say v)in the path of order n. This edge dominates all edges which are adjacent to the vertex v and do not belong to the path graph. Therefore, the number of these edges is n. The remained edges, that are not adjacent by these edges, are the edges of the path graph. Now, all edges in the thorn graph that belong to the path graph have an even degree. Accordingly, all three consecutive edges can be dominated by one edge. Thus, the minimum number of edges that dominates the edges of the path graph is $\left\lceil \frac{n}{3} \right\rceil$. Therefore, $\gamma'_{es}(P_{n,r,s}) = n + \left\lceil \frac{n}{3} \right\rceil$.

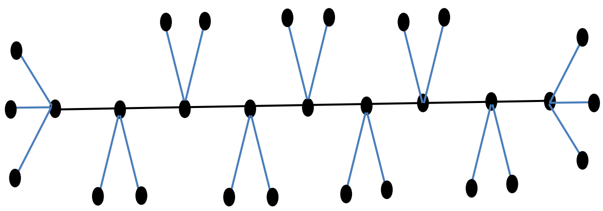

Figure 1. The thorn graph $P_{9,2,3}$

Case 2. If $s\ \ and\ r$ are odd, then all terminal edges in the thorn graph that are adjacent to the pendants vertices of the path graph have odd degree in addition to the pendants vertices of path. Thus, the pendant vertices must belong to the minimum ESEDS. These two vertices dominate the terminal edges in the thorn graph that adjacent to it (as an example, see Figure 2).

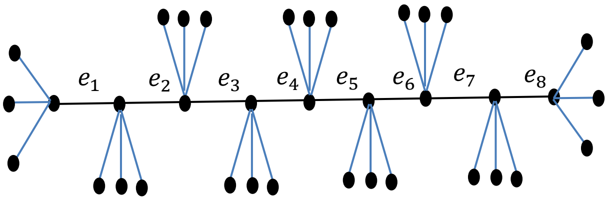

Figure 2. The thorn graph $P_{9,3,3}$

All remained edges of the thorn graph have even degree, so the set $D_1 = \left\{ e_{3+2k}, k = 0,1, \dots, \left\lceil \frac{n-4}{2} \right\rceil \right\}$ where all these edges belong to the path graph (as an example, see Figure 2). The set D_1 is the minimum ESEDS of the remain vertex that not dominate by the edges e_1 and e_n and the number vertices of the set D_1 is $\left\lceil \frac{n-2}{2} \right\rceil$. Thus, $\gamma'_{es}(P_{n,r,s}) = 2 + \left\lceil \frac{n-4}{2} \right\rceil$.

Case 3. If s *is even and* r *is odd*, then all edges in the thorn graph have even degree (as an example, see Figure 1).

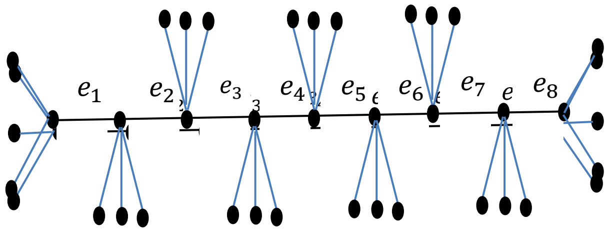

Figure 3. The thorn graph $P_{9,3,2}$

The pendant edges of the path graph (as an example, e_1 *and* e_8 in the Figure 3) must belong to the minimum ESEDS to dominate the terminal edges that adjacent to it. In same technique in previous case the set D_1 is minimum ESEDS to other edges in the thorn graph. Thus, $\gamma'_{es}(P_{n,r,s}) = 2 + \left\lceil \frac{n-4}{2} \right\rceil$.

Case 4. If s *and* r are even, the terminal edges in the thorn graph that adjacent to the pendant vertices in the path graph have even degree and the pendant vertices of the path have odd degree, thus one edge from the terminal edges must be taken in the minimum ESEDS (as an example, see the Figure 4).

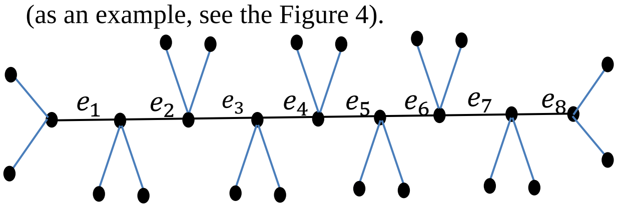

Figure 4. The thorn graph $P_{9,2,2}$

This edge dominates all other terminal edges that adjacent to it. The same technique used in the other terminal, sine for each vertex in the path graph, all terminal edges that adjacent to it have odd degree. Also, the edges of path graph except the pendant have even graph. Thus, again one edge from the terminal edges of a vertex in the path graph must be taken in the minimum ESEDS. The remained vertices which no dominate by the select previous edges are the vertices of path graph except the terminal edges. Therefore, the minimum number of edges that dominate the remained edges is $\left\lceil \frac{n-2}{3} \right\rceil$. Thus, $\gamma'_{es}(P_{n,r,s}) = n + \left\lceil \frac{n-2}{3} \right\rceil$.

From the cases above, the result is obtained.

Proposition .4: Let G be a thorn rod path $P_{n,m}$. Then

$$\gamma'_{es}(P_{n,m}) = \begin{Bmatrix} 1, & if\ n = 2\ and\ m\ is\ even \\ 3, & if\ n = 2\ and\ m\ is\ odd \\ 4 + \left\lceil \frac{n-3}{3} \right\rceil, & if\ n > 2\ and\ m\ is\ even \\ 2 + \left\lceil \frac{n-1}{3} \right\rceil, & if\ n > 2\ and\ m\ is\ even \end{Bmatrix}.$$

Proof. The proof is treated separately for four cases.

Case 1. If $n = 2$ and m is even, then all edges of the thorn rod graph are even degree, so the edge of path dominates all other edges. Thus, $\gamma'_{es}(P_{2,m}) = 1$.

Case 2. If $n = 2$ and m is odd, then all terminal edges of the thorn rod graph have odd degree and the edge of path has even degree. Thus, one of terminal edges must be taken in the minimum ESEDS from each side of the path. Also, the edge of P_2 must be taken according to observation 5. Thus, $\gamma'_{es}(P_{2,m}) = 3$.

Case 3. If $n > 2$ and m is even, then all terminal edge of the thorn rod path graph has even degree. In addition, the pendant edges of the path graph have odd degree. Finally, the other edges of path graph have even degree (as an example, see the Figure 5).

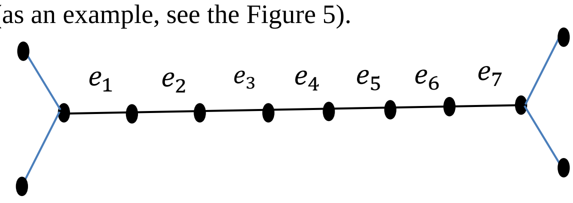

Figure 5. The thorn rod graph $P_{n,2}$

In each side of the path, one each must be taken to add to the minimum ESEDS to dominate all terminal edges in the thorn rod path graph. Also, the terminal edges of the path graph (as an example, e_1 and e_8 in the Figure 5) must be taken in the minimum ESEDS, according to Observation 5. The remained edges which are not dominated by the four vertices are $\{e_2, e_3, \ldots, e_{n-1}\}$ of the path graph. All these edges have even degree, so all edges in thorn graph that belong to the path graph have even degree. Therefore, all three consecutive edges can be dominated by one edge and $\gamma'_{es}(P_{2,m}) = 4 + \left\lceil \frac{n-3}{3} \right\rceil$.

Case 4. If $n > 2$ and m is odd, then all terminal edge of the thorn rod path graph has odd degree. In addition, all the edges of the path graph have even degree (as an example, see Figure 6).

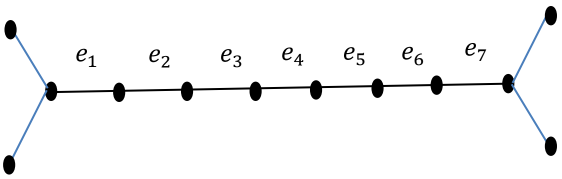

Figure 6. The thorn rod graph $P_{n,3}$

In each side of the path, each one must be taken to add to the minimum ESEDS to dominate all terminal edges in the thorn rod path graph. The remained edges not dominated by the two vertices are $\{e_1, e_2, \ldots, e_n\}$ of the path graph. All these edges have even degree, so all edges in thorn graph that belong to the path graph have even degree. Therefore, all three consecutive edges can be dominated by one edge and $\gamma'_{es}(P_{2,m}) = 2 + \left\lceil \frac{n-1}{3} \right\rceil$.

From the cases above, the result is obtained.

3. APPLICATIONS

In this paper, many applications are stated to domination. As in the case of the main subject applications, this subject has specialized applications such as its use in inspection of two types or any application that requires two classes of work. To obtain an even number from adding two numbers, they are either both even or both odd. Domination in graphs has applications to several fields like school bus routing, computer communication networks, locating radar stations problem, nuclear power plants problem, modeling biological networks, modeling social networks, facility location problems, and coding theory. For more details about the applications in domination the reader can see [14].

4. CONCLUSIONS

Throughout this paper a new definition of domination in graphs is introduced which is called even sum domination. The results obtained in this study depended on the edge set.

For the edge set many properties and bounded are determined. For some certain graphs such as path, thorn, and thorn rod, this number is calculated with giving an algorithm for path. In future we extend this work with algorithms to calculate this number for more certain graphs.

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Arabic Abstract

أدت التطبيقات المختلفة للهيمنة والبحوث الحديثة حول أنواع مختلفة من الهيمنة في المخططات إلى إثارة اهتمامنا باستكشاف نوع خاص جديد من الهيمنة في المخططات وذلك ما قمنا به في هذا البحث. إن معظم البحوث التي تمت دراسنها في الهيمنة تركز بشكل أكبر على تحديد الشروط على المجموعة المهيمنة للتوصل إلى مفهوم جديد للهيمنة. في هذا البحث، قمنا بتقديم المعلمة الجديدة للهيمنة في المخطط والتي تسمى مجموعة سيطرة الحافة ذات المجموع الزوجي .(ESEDS) وقمنا بمناقشة العديد من خصائص هذه الأعداد. علاوة على ذلك، قدمنا المفهوم الجديد لمخطط المسار وما يرتبط به مثل مخطط مسار الشوكة ومخطط مسار الشوكة ومسار قضيب الشوك. كما وضعنا خوارزمية للحصول على مجموعة هيمنة الحافة ذات المجموع الزوجي لمخطط المسار.