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Efficient Iterative Transform Approach for Solving Time-Fractional Fokker-Planck Equations

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ABSTRACT

An iterative semi-analytical transform approach is suggested in this paper for solving a time-fractional Fokker-Planck (FrF-P) partial differential equations. The Kashuri-Fundo transform and the variational iteration method are the key components of the suggested method. The fractional derivative is taken in the Caputo sense. The solution is given as a rapidly converging fractional power series with simple coefficients. Some illustrative examples are solved to show how practical and effective the proposed approach is.

1. INTRODUCTION

Partial differential equations (PDEs) are frequently utilized to expressing engineering and natural activities in the various fields of physics, chemistry, biology and applied mathematics. A variety of nonlinear partial differential equations (NLPDEs) have been the focus of important studies by physicists, mathematicians, and scientists in the past decades.

Finding the solution to partial NLDEs is challenging because to their nonlinear components. Although obtaining approximate or exact solutions to nonlinear partial differential equations (NLPDEs) is essential in many study areas, it remains a difficult task that require the development of new methodologies. In order to get an analytical solution, dependable and effective approaches must be developed [1]. The exact solution of these DEs is significant since many practical sciences, including quantum mechanics, hydrodynamics, plasma physics, and nonlinear optics, depend on the ability of predicting the future behavior of a dynamic system.

In recent years, fractional calculus, which is seen as a generalization of standard integer-order integration and differentiation, has received a lot of attention due to the wide range of disciplines in which it is used in modern life. Fractional derivatives have been defined via many suggested definitions including Riesz, Riemann-Liouville, Grunwald-Letnikov, Caputo, and conformable fractional definitions [2]-[4]. Fractional partial differential equations (FrPDEs), are utilized for modeling wide range of real-life applications. The FrPDEs gained importance and popularity because of their wide applications across many fields, including quantum physics, electrical circuits, and theoretical biology [5,6].

A significant amount of research has been done to find solutions for the FrDEs [7-11] and references therein. However, it can be difficult to find exact analytical solutions to the majority of these equations because of the complexity of nonlinear components and fractional derivatives; as a result, approximation and numerical approaches are acceptable for handling the issue. Thus, many iterative and hybrid methods have been proposed, such as the homotopy perturbation method (HPM) [12], the generalized differential transform method [13], the fractional variational iteration method (FrVIM) [14], the Adomian decomposition method (ADM) [15], the homotopy perturbation Sumudu transform method [16], the Kashuri Fundo transform and homotopy perturbation method [17] and references therein.

One of the most well-known and significant equations in the fields of statistical physics, natural science is the Fokker-Planck (F-P) equation. It was first proposed by Fokker and Planck to explain the Brownian motion of particles and the change in probability of a random function in space and time [18].

The objective of this study is to propose an iterative semi-analytic transform approach to approximate the solution of a time-fractional Fokker-Planck (FrF-P) partial differential equations. The suggested approach is termed the fractional Kashuri Fundo variational homotopy method (Fr-KFVHM), it is a combination of

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the Kashuri-Fundo transform (KFT) [19], the VIM [20], and the HPM [21]. The Fr-KFVHM helps in avoiding the complications that often arise when trying to find the Lagrange multiplier (LagM) and the complex integrations that are employed in VIM.

2. OVERVIEW OF THE FOKKER-PLANCK (F-P) EQUATION

The following equation represents the general F-P equation [18]:

$$\frac{\partial z(x,t)}{\partial t} = \left[-\frac{\partial}{\partial x}Q(x) + \frac{\partial^2}{\partial x^2}R(x)\right] z(x,t) \qquad (1)$$

subject to,

$$z(x,0) = f(x)\,, \quad x \in \mathbb{R}\,, \qquad (2)$$

The drift and the diffusion coefficients are $Q(x)$ and $R(x)$, respectively. The diffusion and drift coefficients might be time-dependent.That is, "(1)" can be written as:

$$\frac{\partial z(x,t)}{\partial t} = \left[-\frac{\partial}{\partial x}Q(x,t) + \frac{\partial^2}{\partial x^2}R(x,t)\right] z(x,t)\,. \qquad (3)$$

Equation (1) represents mathematically a linear second order PDE of parabolic type. The following equation is a generalized version of "(1)" for M variables $x_1, x_2, \dots x_M$:

$$\frac{\partial z(\boldsymbol{x},t)}{\partial t} = \left[-\sum_{r=1}^{M}\frac{\partial}{\partial x_r}Q_r(\boldsymbol{x}) + \sum_{r,s=1}^{M}\frac{\partial^2}{\partial x_r \partial x_s}R_{r,s}(\boldsymbol{x})\right] z(\boldsymbol{x},t) \qquad (4)$$

subject to,

$$z(\boldsymbol{x},0) = f(\boldsymbol{x})\,, \boldsymbol{x} = (x_1, x_2, \dots x_M) \in \mathbb{R}^M \qquad (5)$$

The nonlinear F-P (NLF-P) equation is the most common type of F-P equations and has significant applications in many fields, including engineering, pattern formation, psychology, neurosciences, population dynamics, nonlinear hydrodynamics, plasma physics, etc. The NLF-P equation is expressed as follows for the one variable case:

$$\frac{\partial z(x,t)}{\partial t} = \left[-\frac{\partial}{\partial x}Q(x,t,z) + \frac{\partial^2}{\partial x^2}R(x,t,z)\right] z(x,t) \qquad (6)$$

3. VARIATIONAL ITERATION METHOD (VIM)

Illustration of the concept VIM based on NLPDE [20] is

$$U\mathrm{z} + T\mathrm{z} = P(x) \qquad (7)$$

where T is nonlinear operator, U is linear operator, and $P(x)$ is an analytical function. Correction functional for "(7)" of VIM is

$$\mathrm{z}_{m+1}(x) = \mathrm{z}_m(x) + \int_0^x \lambda(\xi)\,[U\mathrm{z}_m(\xi) + T\mathrm{z}_m(\xi) - P(\xi)]d \qquad (8)$$

The (LagM) $\lambda(\xi)$ may be determined by the variational theory. A restricted variation is z_m, i.e. $\delta \mathrm{z}_m$ = 0 and the mth approximation is denoted by the index m.

The approximation $\mathrm{z}_{m+1},\ m \geq 0$ of z cab be computed by any selective function z_0 and using LagM. To determine $\lambda(\xi)$, integration by parts may be used; and the solution is given by,

$$z = \lim_{m\to\infty} \mathrm{z}_m \qquad (9)$$

4. KASHURI AND FUNDO TRANSFORM

Let Ω be a set of functions of exponential order [19],

$$\Omega = \left(\mathscr{k};\ |\mathscr{k}(t)| \leq N e^{\frac{|t|}{(k_j)^2}}\,, t \in (-1)^j \times, [0,\infty)\right) \qquad (10)$$

where $N, k_1, k_2 > 0$.

The KFT is defined as the following and denoted by the operator (:) ,

$$\boldsymbol{\mathcal{K}}[\mathscr{k}(t)](\mathscr{w}) = \frac{1}{v}\int_0^\infty \mathscr{k}(t) e^{\frac{-t}{\mathscr{w}^2}} dt = \mathcal{A}(\mathscr{w}) \qquad (11)$$

where $t \geq 0\,;\ -k_1 < w < k_2$.

Let $\mathcal{A}(\mathscr{w})$ be the KFT of $\mathscr{k}(t)$. The fundamental properties of KFT are [25]

$$1.\,\boldsymbol{\mathcal{K}}[\mathscr{k}'(t)](\mathscr{w}) = \frac{\mathcal{A}(\mathscr{w})}{\mathscr{w}^2} - \frac{\mathscr{k}(0)}{\mathscr{w}} \qquad (12)$$

$$2.\,\boldsymbol{\mathcal{K}}[\mathscr{k}''(t)](\mathscr{w}) = \frac{\mathcal{A}(\mathscr{w})}{\mathscr{w}^4} - \frac{\mathscr{k}(0)}{\mathscr{w}^3} - \frac{\mathscr{k}'(0)}{\mathscr{w}} \qquad (13)$$

3. $\mathcal{K}[k^{(n)}(t)](w) = \frac{\mathcal{A}(w)}{w^{2n}} - \sum_{k=0}^{n-1} \frac{k^{(k)}(0)}{w^{2(n-k)-1}}$ (14)

TABLE 1. The typical kft for some functions [19]

$\boldsymbol{k(t)}$	$\boldsymbol{\mathcal{K}[k(t)](w) = \mathcal{A}(w)}$
1	w
$\boldsymbol{t^n}$ **,** $\boldsymbol{n \geq 0}$	$n!\, w^{2n+1}$
$\boldsymbol{e^{-\mu t}}$	$\frac{w}{1+\mu w^2}$
$\boldsymbol{sin(\mu t)}$	$\frac{a w^2}{1+\mu^2\, w^4}$
$\boldsymbol{cos(\mu t)}$	$\frac{w^2}{1+\mu^2 w^4}$

Theorem 1. [22]. The KFT of the Riemann-Liouville fractional integral $\mathfrak{Q}_t^{\alpha} z(x,t)$ and the Caputo fractional derivative $D_t^{\alpha} z(x,t)$ is given by

i. $\mathcal{K}\{\mathfrak{Q}_t^{\alpha} z(x,t)\} = w^{2m} \mathcal{A}(x,w)$,

ii. $\mathcal{K}\{D_t^{\alpha} z(x,t)\} = \frac{\mathcal{A}(x,w)}{w^{2m}} - \sum_{k=0}^{m-1} \frac{1}{w^{2(\alpha-k)-1}} \frac{\partial^k z(x,0^+)}{\partial t^k}$

where $m-1 < \alpha < m \in N$.

iii. $\mathcal{K}\left\{\frac{t^{n\alpha}}{\Gamma(1+m\alpha)}\right\} = w^{2m\alpha+1}$

5. HOMOTOPY PERTURBATION METHOD

The basic idea of the HPM is explained by considering the following nonlinear system[21] ,

$$U(z) + T(z) - g(s) = 0\,, \quad s \in \boldsymbol{\Phi} \qquad (15)$$

$$B\left(z, \frac{\partial z}{\partial m}\right) = 0\,, \quad s \in \boldsymbol{\Pi} \qquad (16)$$

where T is nonlinear operator, U is linear operator, and $g(s)$ is an analytical function.

The Homotopy technique for "(15)" is,

$$w(r,p): \boldsymbol{\Phi} \times [0,1] \to \mathbb{R} \qquad (18)$$

satisfying,

$$\mathfrak{U}(w,p) = (1-p)[U(w) - U(z_0)] + P[U(w) + T(w) - g(s)] = 0, \quad s \in \boldsymbol{\Phi} \qquad (19)$$

$\mathbb{R}$ is the real numbers, $p \in [0,1]$ increases from 0 to 1, and z_0 is initial approximate solution of "(19)" satisfying the boundary conditions "(15)". Obviously, from "(19)", we have

$$\mathfrak{U}(w,0) = U(w) - L(z_0) = 0 \qquad (20)$$

$$\mathfrak{U}(w,1) = U(w) + T(w) - g(s) = 0 \qquad (21)$$

Suppose that the solution of "(19)" can be written as a power series in p:

$$w = w_0 + p w_1 + p^2 w_2 + \cdots \qquad (22)$$

The solution z of (19), by Setting $p = 1$ is,

$$z = \lim_{p \to 1} w = w_0 + w_1 + w_2 + \cdots \qquad (23)$$

For most cases, "(23)" convergent, however the nonlinear operator affects a rate of convergence.

6. FRACTIONAL VARIATIONAL HOMOTOPY TRANSFORM METHOD (Fr-VHTM)

Fr-VHTM is combined from the KFT, VIM, and HPM. The method begins by applying the KFT for both sides of a given DE. The resulting equation will be multiplied by the LagM to generate the recurrence relation. Then, the recurrence relation is limited to determine the LagM. The technique is significant since it does not require the integral part or the convolution theorem neither the convolution theorem nor the integral part used in VIM.

Applying KFT of "(7)", yields,

$$\mathcal{K}[Uz + Tz - P(x)] = 0 \qquad (24)$$

Multiplying (24) by LagM $\lambda(w)$, we get

$$\lambda(w)\mathcal{K}[Uz + Tz - P(x)] = 0. \qquad (25)$$

The recurrence relation to calculate the LagM is,

$$z_{m+1}(x,w) = z_m(x,w) + \lambda(w)\mathcal{K}[Uz + Tz - P(x)] \qquad (26)$$

The optimality criterion is employed to calculate the LagM $\lambda(w)$ by using the KFT and

$$\frac{\delta z_{m+1}(x,w)}{\delta z_m(x,w)} = 0. \qquad (27)$$

Then $\lambda(w) = -w^{2\alpha}$. By using the value of LagM and the inverse of KFT in "(26)", we obtain the approximate solution.

$$z_{m+1}(x,w) = z_m(x,w) + \mathcal{K}^{-1}\left[-w^{2\alpha}\mathcal{K}[Uz + Tz - P(x)]\right],$$

$m = 0,1,2,3, \ldots$ (28)

The HPM can be expressed as follows for nonlinear terms,

$$T(z) = \sum_{j=0}^{\infty} p^j \mathrm{H}_j = \mathrm{H}_0 + p\mathrm{H}_1 + p^2\mathrm{H}_2 + \cdots \qquad (29)$$

where $H_m{}'$ s denote the He's polynomials.

$$H_m(z_0+z_1+z_2+\cdots+z_m) = \frac{1}{m!}\frac{\partial^m}{\partial p^m}\left[T\left(\sum_{j=0}^{\infty} p^j z_j\right)\right]_{p=0}, \quad m=0,1,2,3,\ldots \quad (30)$$

The following approximations can be found,

$$p^0 = z_0(x,t) \quad (31)$$

$$p^1 = z_1(x,t) = -\mathcal{K}^{-1} \quad (32)$$

$$p^2 = z_2(x,t) = -\mathcal{K}^{-1}\left[w^{2\alpha}\mathcal{K}\left[T\left(z_1(x,t)\right) - H\left(z_1(x,t)\right)\right]\right] \quad (33)$$

$$p^3 = z_3(x,t) = -\mathcal{K}^{-1}\left[w^{2\alpha}\mathcal{K}\left[T\left(z_2(x,t)\right) - H\left(z_2(x,t)\right)\right]\right] \quad (34)$$

and so on.

$$z_m(x,t) = z_0+z_1+z_2+z_3+\cdots \quad (35)$$

7. APPLICATIONS

In number of practical scientific disciplines, including physics, engineering, the life sciences, and statistics, the Fokker-Planck equation has been considered as one of the most significant differential equations. The random motion of infinitesimally small particles in a changing medium is studied using the Fokker-Planck equation, and statistical processes and reactions in systems that rely on scattering and atomic activity are also examined. It is also a powerful tool for analyzing how materials are transported and how gases and liquids are distributed in engineering systems. To investigate heat transfer processes and the probability distribution of tiny particles, like electrons and photons, in electromagnetic systems, for instance, physicists utilize the Fokker-Planck equation. This equation is used in engineering to study the movement of matter and energy in physical and chemical systems. It is used to study biological reaction models and the activity of chemicals in living cells in the life sciences. The Fokker-Planck equation is additionally used in disciplines including statistics, economics, and finance. It is used to examine trends in stock price changes and market movements. In general, depending on the scientific field and the phenomenon being studied, the Fokker-Planck equation has a wide range of applications, whether in its form with integer derivatives or with fractional derivatives.

Example 1:

$$\frac{\partial^\alpha}{\partial t^\alpha} = \left[-\frac{\partial}{\partial x}x + \frac{\partial^2}{\partial x^2}\frac{x^2}{2}\right]z$$

$$z(x,0) = x \quad , \quad x \in [0,2]\,, t \geq 0\,,\ 0 < \alpha \leq 1$$

$$\left[\frac{\partial^\alpha}{\partial t^\alpha} + \left[\frac{\partial}{\partial x}x - \frac{\partial^2}{\partial x^2}\frac{x^2}{2}\right]z\right] = 0 \quad (36)$$

The recurrence relation after applying the KFT for both sides of "(36)" , then multiplying by $\lambda(w)$ is, we obtain

$$z_{m+1}(x,w) = z_m(x,w) + \lambda(w)\left[K\left[\frac{\partial^\alpha}{\partial t^\alpha} + \left[\frac{\partial}{\partial x}x - \frac{\partial^2}{\partial x^2}\frac{x^2}{2}\right]z\right]\right] \quad (37)$$

Using the KFT after considering the variation of "(37)" w.r.t the independent variable z_m.

$$\delta z_{m+1}(x,w) = \delta z_m(x,w) + \lambda(w)\left[\left(\frac{1}{w^{2\alpha}}\delta z_m(x,w) - z(x,0) + K\left[\left[\frac{\partial}{\partial x}x - \frac{\partial^2}{\partial x^2}\frac{x^2}{2}\right]z\right]\right)\right]$$

$$\delta z_{m+1}(x,w) = \delta z_m(x,w)\left(1 + \frac{1}{w^{2\alpha}}\lambda(w)\right)$$

When $\frac{\delta z_{m+1}}{\delta z_m} = 0$,

$(\frac{1}{w^2}\lambda(w) = -1) \times w^{2\alpha}$, then

$\lambda(w) = -w^{2\alpha}$

$$z_{m+1}(x,w) = z_m(x,w) - w^{2\alpha}\left[K\left[\frac{\partial^\alpha}{\partial t^\alpha} + \left[\frac{\partial}{\partial x}x - \frac{\partial^2}{\partial x^2}\frac{x^2}{2}\right]z\right]\right]$$

Taking the inverse of the KFT

$$z_{m+1}(x,w) = z_m(x,w) - K^{-1}\left[w^{2\alpha}\left[K[\frac{\partial^\alpha}{\partial t^\alpha} + \left[\frac{\partial}{\partial x}x - \frac{\partial^2}{\partial x^2}\frac{x^2}{2}\right]z]\right]\right]$$

The HMP can be applied to have

$$z_0 + pz_1 + p^2z_2 + \cdots = z_m(x,t) + pK^{-1}\left[w^{2\alpha}\left[K\left[\left(\left[\left[-\frac{\partial}{\partial x}x + \frac{\partial^2}{\partial x^2}\frac{x^2}{2}\right]\right]z_0\right) + p\left(\left[-\frac{\partial}{\partial x}x + \frac{\partial^2}{\partial x^2}\frac{x^2}{2}\right]z_1\right) + p^2\left(\left[-\frac{\partial}{\partial x}x + \frac{\partial^2}{\partial x^2}\frac{x^2}{2}\right]z_2\right) + \cdots\right]\right]\right] \quad (38)$$

$$p^0: z_0 = z_0(x,t) = x\,,$$

$$p^1: z_1 = K^{-1}\left[w^{2\alpha}\left[K\left[-\frac{\partial}{\partial x}x + \frac{\partial^2}{\partial x^2}\frac{x^2}{2}\right]z_0\right]\right] = K^{-1}[w^{2\alpha+1}x] = x\frac{t^\alpha}{\Gamma(\alpha+1)},$$

$$p^2: z_2 = K^{-1}\left[w^{2\alpha}\left[K\left[-\frac{\partial}{\partial x}x + \frac{\partial^2}{\partial x^2}\frac{x^2}{2}\right]z_1\right]\right]$$

$= K^{-1}[w^{4\alpha+1}x] = x\frac{t^{2\alpha}}{\Gamma(2\alpha+1)},$

$p^3: z_3 = K^{-1}\left[w^2 \left[K\left[-\frac{\partial}{\partial x}x + \frac{\partial^2}{\partial x^2}\frac{x^2}{2}\right]z_2\right]\right]$

$= K^{-1}[w^{6\alpha+1}x] = x\frac{t^{3\alpha}}{\Gamma(3\alpha+1)},$

$p^m: z_m = x\frac{t^{m\alpha}}{\Gamma(m\alpha+1)} = x\left(1 + \frac{t^{\alpha}}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \cdots\right) = xE_{\alpha}(t^{\alpha})$ (39)

TABLE 2. The 10th fr-vhtm approximate solutions of Example 1 at α=1

t	**Exact solution**	**Approximation solution**	**Absolute error**
0.1	1.1051709180756477	1.1051709180756475	0.222045×10^{-15}
0.2	1.2214027581601699	1.2214027581601692	0.666134×10^{-15}
0.3	1.3498588075760032	1.3498588075759577	0.455191×10^{-13}
0.4	1.4918246976412703	1.4918246976401834	0.108691×10^{-11}
0.5	1.6487212707001282	1.6487212706873657	0.176250×10^{-10}
0.6	1.8221188003905090	1.8221188002948574	0.956517×10^{-10}
0.7	2.0137527074704766	2.0137527069445813	0.525895×10^{-9}
0.8	2.2255409284924680	2.2255409261876826	0.230479×10^{-8}
0.9	2.4596031111569500	2.4596031026621 000	0.849485×10^{-8}

Example 2:

$\frac{\partial^{\alpha}}{\partial t^{\alpha}} = \left[-\frac{\partial}{\partial x}\frac{x}{6} + \frac{\partial^2}{\partial x^2}\frac{x^2}{12}\right]z$ (40)

$x \in [0,2]\,, t \geq 0\,,\ 0 < \alpha \leq 1$

$z(x,0) = x^2$ The recurrence relation after applying the KFT for both sides of "(40)", then multiplying by $\lambda(w)$ is, we conclude that

$z_{m+1}(x,w) = z_m(x,w) + \lambda(w)\left[K\left[\frac{\partial^{\alpha}}{\partial t^{\alpha}} + \left[\frac{\partial}{\partial x}\frac{x}{6} - \frac{\partial^2}{\partial x^2}\frac{x^2}{12}\right]z\right]\right]$ (41)

Applying the same steps as in Example1, we have $\lambda(w) = -w^{2\alpha}$, and

$p^0: z_0 = z_0(x,t) = x^2,$

$p^1: z_1 = K^{-1}\left[w^{2\alpha}\left[K\left[-\frac{\partial}{\partial x}\frac{x}{6} + \frac{\partial^2}{\partial x^2}\frac{x^2}{12}\right]z_0\right]\right]$

$= \frac{x^2}{2}\frac{t^{\alpha}}{\Gamma(\alpha+1)}$

$p^2: z_2 = K^{-1}\left[w^{2\alpha}\left[K\left[-\frac{\partial}{\partial x}\frac{x}{6} + \frac{\partial^2}{\partial x^2}\frac{x^2}{12}\right]z_1\right]\right]$

$p^3: z_3 = K^{-1}\left[w^2\left[K\left[-\frac{\partial}{\partial x}\frac{x}{6} + \frac{\partial^2}{\partial x^2}\frac{x^2}{12}\right]z_2\right]\right]$

$= K^{-1}\left[w^{6\alpha+1}\frac{x^2}{8}\right] = \frac{x^2}{8}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)},$

$p^m: z_m = \frac{x^2}{2^m}\frac{t^{m\alpha}}{\Gamma(m\alpha+1)}$

$= x^2\left(1 + \frac{t^{\alpha}}{2\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{2^2\Gamma(2\alpha+1)} + \frac{t^{3\alpha}}{2^3\Gamma(3\alpha+1)} + \cdots\right) = \frac{x^2}{4}\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}$

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Arabic Abstract

تم اقتراح طريقة تحويل شبه تحليلية تكرارية في هذه الورقة لحل معادلات فوكر-بلانك التفاضلية الجزئية للزمن الكسرية .(FrF-P) يعد تحويل Kashuri-Fundo وطريقة التكرار المتغير المكونات الرئيسية للطريقة المقترحة. يتم أخذ المشتق الكسري بمعنى كابوتو. يتم تقديم الحل على شكل سلسلة قوى كسرية متقاربة بسرعة مع معاملات بسيطة. تم حل بعض الأمثلة التوضيحية لتوضيح مدى عملية وفعالية النهج المقترح.