

Research Article

New Types of Fuzzy Ideals in KU-algebra

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Abstract

We introduce in this research, new types of fuzzy ideals of an algebra of type KU : Fuzzy Complete Ideal, Fuzzy E-Ideal and Fuzzy Complete E-Ideal, respectively. In addition, some propositions explained relationships with these fuzzy ideals types.

1. Introduction

In [10] , Leerawat and Prabpayak introduced congruences and ideals in algebras of type BCC [1,6], also studied and introduced a new algebraic structure which is denoted by algebra of type KU or KU-algebra, also investigated some related properties. Jawad and AL-Shaher [7] studied new types of ideals of KU-algebras, and also stated the relationship between them. Zadeh [13] introduced the notion of fuzzy sets. Dudek et al studied fuzzy ideals, also several fuzzy structures in BCC-algebras are considered

2. Basic Concepts and Notations

In this section, we introduce definitions in KU-g , B.D KU-involutory , complete ideal ,

Definition (2.1)[10, 11]

Algebra $(Y, \diamond, 0)$ with type $(2, 0)$ is called KU-algebra and we will denote by KU-g , if it hold the axioms:

- (1) $(r \diamond s) \diamond ((s \diamond t) \diamond (r \diamond t)) = constant\ 0, \forall r, s, t \in Y$
- (2) $r \diamond 0 = constant\ 0$
- (3) $constant\ 0 \diamond r = r$
- (4) $r \diamond s = constant\ 0$ and $s \diamond r = constant\ 0$ implies $r = s$

Theorem (2.2)[8]

In any KU-g Y , then the following hold:
 $\forall r, z, t \in Y$

- (1) $r \lesssim z$ imply $z \diamond t \lesssim r \diamond t$

Definition (2.3)[12]

If an element "e" of a KU-g Y hold $r \lesssim e$ for all $r \in Y$, then the element "e" is said to be a unit in Y . A KU-g with unit is denoted by the bounded (or by B.D)

Lemma (2.4)[7]

Proposition (2.5)[7]

In any B.D KU-g Y , the following are hold:
 For any $k, s \in Y$,

- (1) $k^{\circ\circ} \lesssim k$

[2,3,4]. Moreover, Mostafa et al [8] studied the notion of fuzzy KU-ideals of KU-algebras and then they investigated several basic properties which are related to fuzzy KU-ideals. Mostafa and Kareem [9] introduced fuzzy n-fold KU-ideals of KU-algebras and obtained some related results. The aim of this paper is to introduce fuzzy complete ideal, fuzzy

E-ideal, fuzzy complete E-ideal in KU-algebras, also some of the basic properties are investigated.

E-ideal , complete E-ideal , fuzzy KU-ideal, fuzzy KU-subalgebra, fuzzy ideal, and some of their properties.

- (5) $r \diamond r = constant\ 0$

And in Y define a binary relation \lesssim by :
 $r \lesssim s$ iff $s \diamond r = constant\ 0$, for all $r, s \in Y$.

Thus a KU-g Y hold the conditions :

- (1) $s \diamond r \lesssim r$
- (2) $constant\ 0 \lesssim r$
- (3) $r \lesssim s, s \lesssim r$ implies $r = s$
- (4) $(s \diamond t) \diamond (r \diamond t) \lesssim (r \diamond s)$

- (2) $((z \diamond r) \diamond r) \diamond r = z \diamond r,$

- (3) $((z \diamond r) \diamond r) \lesssim z$

- (4) $r \diamond (z \diamond t) = z \diamond (r \diamond t)$

In B.D KU-g Y , we denote $r \diamond e$ by r° for any $r \in Y$.

It is easy to see that $e^{\circ} = constant\ 0$, $0^{\circ} = e$.

The unit of B.D KU-g is a unique.

- (2) $k \diamond s^{\circ} = s \diamond k^{\circ}$

- (3) $k^{\circ} \diamond s^{\circ} \lesssim s \diamond k,$

- (4) If $s \lesssim k$ implies $k^{\circ} \lesssim s^{\circ}$

- (5) $e^{\circ} \lesssim k$

Definition (2.6)[11] Let \mathcal{A} be a nonempty subset of a KU-g Y . Then \mathcal{A} is said to be a

KU-subalgebra of Y , if $r \diamond s \in \mathcal{A}$, whenever $r, s \in \mathcal{A}$.

Definition (2.7)[10, 11]

Let \mathcal{A} be a nonempty subset from a KU-g Y . Then \mathcal{A} is said to be an ideal from Y , if

- (1) constant $0 \in \mathcal{A}$
- (2) $\forall r, s \in Y$, if $s \diamond r \in \mathcal{A}$ and $s \in \mathcal{A}$ imply $r \in \mathcal{A}$.

Proposition (2.8)[10] Every ideal in KU-g is KU-subalgebra.

Definition (2.9)[8]

Let \mathcal{A} be a nonempty subset from a KU-g Y . Then \mathcal{A} is said to be a KU-ideal from Y , if

- (1) constant $0 \in \mathcal{A}$
- (2) $\forall r, s, t \in Y$, if $r \diamond (s \diamond t) \in \mathcal{A}$ and $s \in \mathcal{A}$ imply $r \diamond t \in \mathcal{A}$.

Definition (2.10)[7]

Let \mathcal{A} be a non-empty subset in KU-g Y is denoted be complete ideal (briefly, c-ideal), if

- (1) constant $0 \in \mathcal{A}$
- (2) $s \diamond k \in \mathcal{A}, \forall s \in \mathcal{A}$ s.t $s \neq$ constant 0 implies $k \in \mathcal{A}$.

Definition (2.11)[7]

Let Y be B.D KU-g. An element $r \in Y$ satisfies $r^\circ = r$, then r is called an

involution. If every element $r \in Y$ is an involution, we call Y is a KU- involutory algebra.

Definition (2.12)[7]

Let \mathcal{A} be a non-empty subset in B.D KU-g Y is called E-ideal if, for all $r \in Y$

- (1) constant $0 \in \mathcal{A}$
- (2) $r \diamond k^\circ \in \mathcal{A}$ and $k \in \mathcal{A}$ implies $r^\circ \in \mathcal{A}$.

Definition (2.13)[7]

Let \mathcal{A} be a non-empty subset in B.D KU-g Y is denoted be complete E-ideal (briefly, c-E-ideal), if

- (1) constant $0 \in \mathcal{A}$
- (2) $r \diamond s^\circ \in \mathcal{A}$, for all $s \in \mathcal{A}$ such that $s \neq$ constant 0 implies $r^\circ \in \mathcal{A}, \forall r \in Y$.

Proposition (2.14)[7]

Every c-ideal of B.D KU-g is c-E-ideal .

Proposition (2.15)[7]

If \mathcal{A} be c-E-ideal in KU-involutory algebra Y , then \mathcal{A} is c-ideal .

Definition (2.16)[13]

Let Y be set and let φ be a fuzzy set (or fuz. set) in Y is define the mapping $\varphi : Y \rightarrow [0,1]$.

If φ and ρ be two fuz. subsets in Y , then $\varphi \subseteq \rho \Leftrightarrow \varphi(r) \leq \rho(r)$, for all $r \in Y$.

Definition (2.17)[5]

If φ and ρ are two fuz. sets in Y . Then
 (1) $(\varphi \cap \rho)(r) = \min\{\varphi(r), \rho(r)\}$, for all $r \in Y$.
 (2) $(\varphi \cup \rho)(r) = \max\{\varphi(r), \rho(r)\}$, for all $r \in Y$
 $\varphi \cap \rho$ and $\varphi \cup \rho$ are fuz. sets of Y .

In general , if $\{\varphi_\alpha : \alpha \in \lambda\}$ is family of fuz. sets of Y , then
 $\bigcap_{\alpha \in \lambda} \varphi_\alpha(r) = \inf\{\varphi_\alpha(r), \alpha \in \lambda\}$, for all $r \in Y$ and
 $\bigcup_{\alpha \in \lambda} \varphi_\alpha(r) = \sup\{\varphi_\alpha(r), \alpha \in \lambda\}$, for all $r \in Y$
 Which are also fuz. sets in Y .

Definition (2.18)[8]

Let Y be KU-g. φ be a fuz. set in Y is denoted be a fuz. KU-subalgebra in Y if it hold , $\forall k, s \in Y$:

- (1) $\varphi (constant 0) \geq \varphi (k)$
- (2) $\varphi (k \diamond s) \geq \min\{ \varphi (k) , \varphi (s) \}$

Definition (2.19)[8]

Let Y be KU-g . A fuz. set φ of Y is called fuz. KU-ideal in Y if it hold , $\forall k, s, t \in Y$:

- (2) $\varphi (k \diamond t) \geq \min\{ \varphi (k \diamond (s \diamond t)) , \varphi (s) \}$

- (1) $\varphi (constant 0) \geq \varphi (k)$

Lemma (2.20)[8] Let φ be fuz. KU-ideal in KU-g Y , if $r \diamond s \lesssim t$ is hold, then $\varphi (s) \geq \min\{ \varphi (r) , \varphi (t) \}$

Definition (2.21)[8]

Let φ be fuz. set of a KU-g Y is called fuz. ideal of Y if, $\forall r, s \in Y$

- (1) $\varphi (constant 0) \geq \varphi (r)$
- (2) $\varphi (r) \geq \min\{ \varphi (s \diamond r) , \varphi (s) \}$

Lemma (2.22)[9]

Let φ be fuz. subset in KU-g Y . Then (1) \cong (2) if :

- (1) φ is fuz. ideal
- (2) φ is fuz. KU-ideal

Lemma (2.23)[9]

Every fuz. KU-ideal in KU-g Y is fuz. KU-subalgebra .

Example (2.25)[9]

Consider $Y = \{0,1,2,3,4\}$ be set as shown in the table:

Remark (2.24)[9] The converse of lemma (2.23) is not true as shown in the example .

\diamond	0	1	2	3	4
0	0	1	2	3	4
1	0	0	0	3	4
2	0	1	0	3	4
3	0	0	0	0	4
4	0	0	0	0	0

Then $(Y, \diamond, constant 0)$ is a KU-g . Define the fuz. set $\varphi : Y \rightarrow [0,1]$ by :

$$\varphi (r) = \begin{cases} 1 & \text{if } r = 0,2,3 \\ 0.5 & \text{if } r = 1,4 \end{cases}$$

Notice that φ is fuz. KU-subalgebra in Y .

But φ is not fuz. KU-ideal in Y , since $\varphi (0 \diamond 1) = 0.5 \not\geq \min\{ \varphi (0 \diamond (3 \diamond 1)) , \varphi (3) \} = 1$

Proposition (2.26)

Let φ be fuz. KU-subalgebra in a KU-g Y , if $k \diamond s \lesssim t$ and the inequality $\varphi (s) \geq \min\{ \varphi (k) , \varphi (t) \}$ hold , $\forall k, s, t \in Y$. Then φ is fuz. KU-ideal of Y .

Proof:

Let φ be a fuz. KU-subalgebra of Y .

Now, by using Definition (2.18) we get $\varphi (constant 0) \geq \varphi (k) , \forall k \in Y$.

Let $k, s \in Y$, from Theorem (2.2)(3) we have $(s \diamond k) \diamond k \lesssim s$, then $\varphi (k) \geq \min\{ \varphi (s) , \varphi (s \diamond k) \}$ (By the condition above) .

So φ is fuz. ideal of Y , this mean φ is fuz. KU-ideal of Y .

3. The Main Results

We provide definitions in this section : fuz. complete ideal , fuz. E-ideal , fuz. complete

Definition (3.1)

Let \mathcal{A} be a c-ideal in B.D KU-g Y . Then the fuz. subset $\varphi_{\mathcal{A}} : Y \rightarrow [0,1]$ is called fuz. complete ideal (briefly, fuz. c-ideal) if , $\forall r \in Y$ and $s \in \mathcal{A}$

E-ideal and study its relationships with fuz. ideal in B.D KU-g .

- (1) $\varphi_{\mathcal{A}}(\text{constant } 0) \geq \varphi_{\mathcal{A}}(r)$
- (2) $\varphi_{\mathcal{A}}(r) \geq \min\{\varphi_{\mathcal{A}}(s \diamond r), \varphi_{\mathcal{A}}(s)\}$

Example (3.2)

Consider the following KU-g Y , [8] with the following table:

\diamond	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	1	0	3	3
3	0	0	2	0	2
4	0	0	0	0	0

Notice that Y is B.D with unit 4.

A subset $\mathcal{A} = \{0,1,2\}$ is c-ideal of Y .

$$\varphi_{\mathcal{A}}(r) = \begin{cases} 0.4 & \text{if } r = 0,1,4 \\ 0.3 & \text{if } r = 2,3 \end{cases}$$

Then $\varphi_{\mathcal{A}}$ is fuz. c-ideal in Y .

$$\text{While } \rho_{\mathcal{A}}(r) = \begin{cases} 0.3 & \text{if } r = 0,1,2,3 \\ 0.2 & \text{if } r = 4 \end{cases}$$

is not fuz. c-ideal in Y , since

If $\varphi_{\mathcal{A}}$ is fuz. set defined as :

$$\rho_{\mathcal{A}}(4) = 0.2 \not\geq \min\{\rho_{\mathcal{A}}(2 \diamond 4), \rho_{\mathcal{A}}(2)\} = 0.3$$

Proposition (3.3)

Every fuz. ideal in B.D KU-g is a fuz. c-ideal .

Proof :

Let \mathcal{A} be a c-ideal , $\varphi_{\mathcal{A}}$ be fuz. ideal of B.D KU-g Y , then by Definition (2.21) we have : $\forall k, s \in Y$

$$(1) \varphi_{\mathcal{A}}(\text{constant } 0) \geq \varphi_{\mathcal{A}}(k)$$

$$(2) \varphi_{\mathcal{A}}(k) \geq \min\{\varphi_{\mathcal{A}}(s \diamond k), \varphi_{\mathcal{A}}(s)\}$$

Since $\mathcal{A} \subseteq Y$, then $\varphi_{\mathcal{A}}(k) \geq \min\{\varphi_{\mathcal{A}}(s \diamond k), \varphi_{\mathcal{A}}(s)\}$, for every $s \in \mathcal{A}$

Thus $\varphi_{\mathcal{A}}$ is fuz. c-ideal of Y .

Remark (3.4)

The conversely of the prop. (3.3) is false as in the next example.

Example (3.5)

In example (3.2), $\varphi_{\mathcal{A}}$ is fuz. c-ideal in Y (when $\mathcal{A} = \{0,1,2\}$), but it 's not fuz. ideal , since

$$\varphi_{\mathcal{A}}(2) = 0.3 \not\geq \min\{\varphi_{\mathcal{A}}(4 \diamond 2), \varphi_{\mathcal{A}}(4)\} = 0.4$$

Corollary (3.6)

Let \mathcal{A} be c-ideal in KU-involutory algebra Y . A fuz. subset $\varphi_{\mathcal{A}}$ is fuz. c-ideal if and only if satisfies

- (1) $\varphi_{\mathcal{A}}(\text{constant } 0) \geq \varphi_{\mathcal{A}}(k)$
- (2) $\varphi_{\mathcal{A}}(k^{\circ\circ}) \geq \min\{\varphi_{\mathcal{A}}(s \diamond k^{\circ\circ}), \varphi_{\mathcal{A}}(s)\}$

Proof:

By Definition (3.1) and Definition (2.11).

Proposition (3.7)

Let \mathcal{A} be c-ideal in B.D KU-g Y and $\{\varphi_{\mathcal{A}_i} : i \in \Delta\}$ be family of fuz. c-ideals , then $\bigcap_{i \in \Delta} \varphi_{\mathcal{A}_i}$ is fuz. c-ideal

Proof :

Let $r \in Y, s \in \mathcal{A}$, then

$$(1) \varphi_{\mathcal{A}_i}(\text{constant } 0) \geq \varphi_{\mathcal{A}_i}(r)$$

$$\inf\{\varphi_{\mathcal{A}_i}(\text{constant } 0)\} \geq \inf\{\varphi_{\mathcal{A}_i}(r)\}.$$

So

$$\bigcap_{i \in \Delta} \varphi_{\mathcal{A}_i}(\text{constant } 0) \geq \bigcap_{i \in \Delta} \varphi_{\mathcal{A}_i}(r)$$

$$(2) \varphi_{\mathcal{A}_i}(r) \geq \min\{\varphi_{\mathcal{A}_i}(s \diamond r), \varphi_{\mathcal{A}_i}(s)\}, \forall s \in \mathcal{A}$$

Thus , $\inf\{\varphi_{\mathcal{A}_i}(r)\} \geq \inf\{\min\{\varphi_{\mathcal{A}_i}(s \diamond r), \varphi_{\mathcal{A}_i}(s)\}\}$

$$\geq \min\{\inf\{\varphi_{\mathcal{A}_i}(s \diamond r)\}, \inf\{\varphi_{\mathcal{A}_i}(s)\}\}$$

So $\bigcap_{i \in \Delta} \varphi_{\mathcal{A}_i}(r) \geq \min\{\bigcap_{i \in \Delta} \varphi_{\mathcal{A}_i}(s \diamond r), \bigcap_{i \in \Delta} \varphi_{\mathcal{A}_i}(s)\}, \forall s \in \mathcal{A}$

Then $\bigcap_{i \in \Delta} \varphi_{\mathcal{A}_i}$ is fuz. c-ideal of Y .

Remark (3.8) In B.D KU-g. Notice that union of two fuz. c-ideals does not necessary

fuz. c-ideal and the following example shows that .

Example (3.9)

Consider $Y = \{0, q, w, h, v, t\}$ and a binary operation \diamond is defined by :

\diamond	0	q	w	h	v	t
0	0	q	w	h	v	t
q	0	0	w	h	v	w
w	0	q	0	h	v	q
h	0	0	0	0	v	0
v	0	0	0	0	0	0
t	0	0	0	h	v	0

Then $(Y, \diamond, \text{constant } 0)$ is a B.D KU-g with unit v [7]. A subset $\mathcal{A} = \{0, q, w, t\}$ is c-ideal of Y . Define fuz. sets $\varphi_{\mathcal{A}}$ and $\rho_{\mathcal{A}}$ from Y into $[0,1]$ by :

$$\varphi_{\mathcal{A}}(r) = \begin{cases} 0.6 & \text{if } r = 0, w, h, v \\ 0.3 & \text{if } r = q, t \end{cases} \quad \text{and}$$

$$\rho_{\mathcal{A}}(r) = \begin{cases} 0.7 & \text{if } r = 0, q, h, v \\ 0.5 & \text{if } r = w, t \end{cases}$$

Then $\varphi_{\mathcal{A}}$ and $\rho_{\mathcal{A}}$ are fuz. c-ideals .

But

$$(\varphi_{\mathcal{A}} \cup \rho_{\mathcal{A}})(r) = \begin{cases} 0.7 & \text{if } r = 0, q, h, v \\ 0.6 & \text{if } r = w \\ 0.5 & \text{if } r = t \end{cases}$$

is not fuz. c-ideal , since

$$(\varphi_{\mathcal{A}} \cup \rho_{\mathcal{A}})(t) = 0.5$$

$$\not\geq \min\{(\varphi_{\mathcal{A}} \cup \rho_{\mathcal{A}})(w \diamond t), (\varphi_{\mathcal{A}} \cup \rho_{\mathcal{A}})(w)\}$$

$$= 0.6$$

Definition (3.10)

A fuz. subset φ in B.D KU-g Y is called fuz. E-ideal, if

$$(1) \varphi(\text{constant } 0) \geq \varphi(k)$$

$$(2) \varphi(k^\circ) \geq \min\{\varphi(k \diamond s^\circ), \varphi(s)\}, \text{ for all } k, s \in Y$$

Note that every fuz. constant in B.D KU-g Y is fuz. E-ideal.

Example (3.11)

Consider $Y = \{0, q, d, h, v\}$ in which the operation \diamond is given by the table:

\diamond	0	q	d	h	v
0	0	q	d	h	v
q	0	0	d	h	h
d	0	q	0	h	v
h	0	0	0	0	q
v	0	0	0	0	0

Then $(Y, \diamond, \text{constant } 0)$ is a B.D KU-g with unit v [7].

If φ and ρ are two fuz. sets defined as :

$$\varphi(r) = \begin{cases} 0.8 & \text{if } r = 0, q, h, v \\ 0.6 & \text{if } r = d \end{cases},$$

$$\rho(r) = \begin{cases} 0.9 & \text{if } r = 0, d, h, v \\ 0.7 & \text{if } r = q \end{cases}$$

Then φ is fuz. E-ideal in Y .

While ρ is not fuz. E-ideal , since

$$\rho(h^\circ) = 0.7 \not\geq \min\{\rho(h \diamond v^\circ), \rho(v)\} = 0.9$$

Proposition (3.12)

Every fuz. ideal in B.D KU-g is fuz. E-ideal

Proof :

Let φ be fuz. ideal in B.D KU-g Y , then by Definition (2.21), we have :

(1) $\varphi(\text{constant } 0) \geq \varphi(k)$

(2) For all $k, s \in Y$, $\varphi(k) \geq \min\{\varphi(s \diamond k), \varphi(s)\}$.

$$\text{Then } \varphi(k^\circ) \geq \min\{\varphi(s \diamond k^\circ), \varphi(s)\} = \min\{\varphi(k \diamond s^\circ), \varphi(s)\},$$

for all $k, s \in Y$ (By Prop. (2.5) (2)).

Thus φ is fuz. E-ideal of Y .

Remark (3.13)

The converse of prop. (3.12) does not hold as in the example .

Example (3.14)

In example (3.11) , φ is fuz. E-ideal of Y , but it's not fuz. ideal in Y , since

$$\varphi(d) = 0.6 \not\geq \min\{\varphi(h \diamond d), \varphi(h)\} = 0.8$$

Proposition (3.15)

Every fuz. E-ideal in KU-involutory algebra Y is fuz. ideal .

Proof :

If φ be a fuz. E-ideal in Y , by Definition (3.10) , we have :

(1) $\varphi(\text{constant } 0) \geq \varphi(r)$

(2) $\varphi(r)$ equal to $\varphi(r^{\circ\circ}) \geq \min\{\varphi(r^\circ \diamond s^\circ), \varphi(s)\}$

$$= \min\{\varphi(s \diamond r^{\circ\circ}), \varphi(s)\} \text{ (By Prop. (2.5) (2))}$$

$$= \min\{\varphi(s \diamond r), \varphi(s)\}, \text{ for all } r, s \in Y$$

(Since Y is a KU-involutory algebra) .

Thus φ is a fuz. ideal of Y .

(3) For any $k, s \in Y$, if $k^\circ \lesssim s$, then $\varphi(k^\circ) \geq \varphi(s)$.

Proposition (3.16)

If φ be a fuz. E-ideal of a B.D KU-g Y , then

(1) $k \in Y, \varphi(k^\circ) \geq \varphi(e)$

(2) $k \in Y, \varphi(k^{\circ\circ}) \geq \varphi(k)$

Proof :

(1) Since φ is fuz. E-ideal , then

$$\varphi(k^\circ) \geq \min\{\varphi(k \diamond e^\circ), \varphi(e)\} = \min\{\varphi(k \diamond \text{constant } 0), \varphi(e)\}$$

$$= \min\{\varphi(\text{constant } 0), \varphi(e)\} = \varphi(e), \forall k \in Y$$

(2) Since φ is fuz. E-ideal , then for all $k \in Y$

$$\varphi(k) = \min\{\varphi(\text{constant } 0), \varphi(k)\}$$

$$= \min\{\varphi(k \diamond k^\circ), \varphi(k)\} \leq \varphi(k^{\circ\circ})$$
 (3) If $k^\circ \lesssim s$ i.e. $s \diamond k^\circ = \text{constant } 0$, then

Proposition (3.17)

Let $\{\varphi_\alpha : \alpha \in \Delta\}$ be family of fuz. E-ideals in B.D KU-g Y , then $\bigcap_{\alpha \in \Delta} \varphi_\alpha$ is fuz. E-ideal of Y .

Proof :

(1) since $\varphi_\alpha(\text{constant } 0) \geq \varphi_\alpha(r)$, $\forall \alpha \in \Delta, \forall r \in Y$, then
 $\inf\{\varphi_\alpha(\text{constant } 0)\} \geq \inf\{\varphi_\alpha(r)\}$.
 So $\bigcap_{\alpha \in \Delta} \varphi_\alpha(\text{constant } 0) \geq \bigcap_{\alpha \in \Delta} \varphi_\alpha(r)$
 (2) let $r, s \in Y$, since $\varphi_\alpha(r^\circ) \geq \min\{\varphi_\alpha(r \diamond s^\circ), \varphi_\alpha(s)\}$, then

$$\varphi(k^\circ) \geq \min\{\varphi(k \diamond s^\circ), \varphi(s)\}, \text{ for any } k, s \in Y \text{ (since } \varphi \text{ is fuz. E-ideal)}$$

$$= \min\{\varphi(s \diamond k^\circ), \varphi(s)\} \text{ (By Prop. (2.5) (2))}$$

$$= \min\{\varphi(\text{constant } 0), \varphi(s)\} = \varphi(s).$$

$$\inf\{\varphi_\alpha(r^\circ)\} \geq \inf\{\min\{\varphi_\alpha(r \diamond s^\circ), \varphi_\alpha(s)\}\}$$

$$\geq \min\{\inf\{\varphi_\alpha(r \diamond s^\circ)\}, \inf\{\varphi_\alpha(s)\}\}$$
 So $\bigcap_{\alpha \in \Delta} \varphi_\alpha(r^\circ) \geq \min\{\bigcap_{\alpha \in \Delta} \varphi_\alpha(r \diamond s^\circ), \bigcap_{\alpha \in \Delta} \varphi_\alpha(s)\}$
 Hence $\bigcap_{\alpha \in \Delta} \varphi_\alpha$ is fuz. E-ideal of Y .

Remark (3.18)

Note that the union of two fuz. E-ideals does not necessarily fuz. E-ideal as in the following example .

Example (3.19)

In example (3.2), notice that :

$$\varphi(r) = \begin{cases} 0.8 & \text{if } r = 0,3 \\ 0.3 & \text{if } r = 1,2,4 \end{cases},$$

$$\rho(r) = \begin{cases} 0.7 & \text{if } r = 0,1,2 \\ 0.4 & \text{if } r = 3,4 \end{cases}$$

are fuz. E-ideals of Y .

$$\text{But } (\varphi \cup \rho)(r) = \begin{cases} 0.8 & \text{if } r = 0,3 \\ 0.7 & \text{if } r = 1,2 \\ 0.4 & \text{if } r = 4 \end{cases}$$

is not fuz. E-ideal, since

$$\begin{aligned}
 (\varphi \cup \rho)(1^\circ) &= 0.4 \\
 &\not\geq \min\{(\varphi \cup \rho)(1 \diamond 2^\circ), (\varphi \cup \rho)(2)\} \\
 &= 0.7
 \end{aligned}$$

(1) $\forall k \in Y, \varphi_{\mathcal{A}}(\text{constant } 0) \geq \varphi_{\mathcal{A}}(k)$

(2) $\forall s \in \mathcal{A}, \varphi_{\mathcal{A}}(k^\circ) \geq \min\{\varphi_{\mathcal{A}}(k \diamond s^\circ), \varphi_{\mathcal{A}}(s)\}$.

Then $\varphi_{\mathcal{A}}$ is fuz. c-E-ideal in Y . While $\rho_{\mathcal{A}}$ is not fuz. c-E-ideal in Y , since

$$\begin{aligned}
 \rho_{\mathcal{A}}(0^\circ) &= 0.5 \\
 &\not\geq \min\{\rho_{\mathcal{A}}(0 \diamond q^\circ), \rho_{\mathcal{A}}(q)\} = 0.8
 \end{aligned}$$

Definition (3.20)

Let \mathcal{A} be c-E-ideal in B.D KU-g Y . The fuz. subset $\varphi_{\mathcal{A}} : Y \rightarrow [0,1]$ is called fuz. complete E-ideal (briefly, fuz. c-E-ideal) if ,

Example (3.21)

In example (3.11) , a subset $\mathcal{A} = \{0, q, d\}$ is c-E-ideal in Y .

Let $\varphi_{\mathcal{A}}$ and $\rho_{\mathcal{A}}$ are fuz. sets defined as the following :

$$\varphi_{\mathcal{A}}(r) = \begin{cases} 0.9 & \text{if } r = 0, d, h \\ 0.6 & \text{if } r = q, v \end{cases}$$

and $\rho_{\mathcal{A}}(r) = \begin{cases} 0.8 & \text{if } r = 0, q, h \\ 0.5 & \text{if } r = d, v \end{cases}$

Proposition (3. 22)

Every fuz. E-ideal in B.D KU-g is fuz. c-E-ideal .

Proof :

Let \mathcal{A} be c-E-ideal in B.D KU-g Y and $\varphi_{\mathcal{A}}$ be fuz. E-ideal of Y , then by Definition (3.10) , we have :

Remark (3. 23)

The converse of prop. (3.22) be not true and the example shows that.

Example (3. 24)

In example (3.21) , $\varphi_{\mathcal{A}}$ is fuz. c-E-ideal in Y , but it's not fuz. E-ideal , since

Proposition (3. 26)

Every fuz. c-ideal of B.D KU-g is a fuz. c-E-ideal .

Proof :

Let $\varphi_{\mathcal{A}}$ be fuz. c-ideal in B.D KU-g Y & \mathcal{A} is c-ideal of Y .

Then \mathcal{A} is c-E-ideal of Y (By Prop. (2.14)).

Since $\varphi_{\mathcal{A}}$ is fuz. c-ideal of Y , from Definition (3.1) , we have :

Remark (3. 27)

In general , not every fuz. c-E-ideal is fuz. c-ideal and the following example shows that.

$$(1) \varphi_{\mathcal{A}} (constant 0) \geq \varphi_{\mathcal{A}} (k)$$

$$(2) \varphi_{\mathcal{A}} (k^{\circ}) \geq \min\{ \varphi_{\mathcal{A}} (k \diamond s^{\circ}) , \varphi_{\mathcal{A}} (s) \} , \text{ for all } k, s \in Y .$$

Since $\mathcal{A} \subseteq Y$, then $\varphi_{\mathcal{A}} (k^{\circ}) \geq$

$$\min\{ \varphi_{\mathcal{A}} (k \diamond s^{\circ}) , \varphi_{\mathcal{A}} (s) \} , \forall s \in \mathcal{A}$$

Thus $\varphi_{\mathcal{A}}$ is fuz. c-E-ideal in Y .

$$\varphi_{\mathcal{A}} (h^{\circ}) = 0.6 \not\geq \min\{ \varphi_{\mathcal{A}} (h \diamond h^{\circ}) , \varphi_{\mathcal{A}} (h) \} = 0.9$$

Corollary (3. 25)

Every fuz. ideal in B.D KU-g is fuz. c-E-ideal .

Proof :

By Proposition (3.12) and Proposition (3.22) .

$$(1) \varphi_{\mathcal{A}} (constant 0) \geq \varphi_{\mathcal{A}} (r) , \forall r \in Y$$

$$(2) \forall s \in \mathcal{A} , \varphi_{\mathcal{A}} (r) \geq \min\{ \varphi_{\mathcal{A}} (s \diamond r) , \varphi_{\mathcal{A}} (s) \}$$

$$\text{Thus } \varphi_{\mathcal{A}} (r^{\circ}) \geq \min\{ \varphi_{\mathcal{A}} (s) , \varphi_{\mathcal{A}} (s \diamond r^{\circ}) \}$$

$$= \min\{ \varphi_{\mathcal{A}} (s) , \varphi_{\mathcal{A}} (r \diamond s^{\circ}) \} , \text{ for every } s \in \mathcal{A}$$

So $\varphi_{\mathcal{A}}$ is fuz. c-E-ideal of Y .

Example (3. 28)

Consider $Y = \{0, q, d, h, v\}$ be set as shown in the following table:

\diamond	0	q	d	h	v
0	0	q	d	h	v
q	0	0	d	d	d
d	0	0	0	q	q
h	0	0	0	0	q
v	0	0	0	0	0

Then $(Y, \diamond, constant 0)$ is a B.D KU-g with unit v [7].

A subset $\mathcal{A} = \{0, q, h\}$ is c-E-ideal and c-ideal (in the same time) of a B.D KU-g Y .

Define the fuz. set $\varphi_{\mathcal{A}} : Y \rightarrow [0,1]$ by :

$$\varphi_{\mathcal{A}} (r) = \begin{cases} 0.8 & \text{if } r = 0, q, d, v \\ 0.2 & \text{if } r = h \end{cases}$$

Then $\varphi_{\mathcal{A}}$ is a fuz. c-E-ideal of Y .

But $\varphi_{\mathcal{A}}$ is not fuz. c-ideal , since

$$\varphi_{\mathcal{A}} (h) = 0.2$$

$$\not\geq \min\{ \varphi_{\mathcal{A}} (q \diamond h) , \varphi_{\mathcal{A}} (q) \} = 0.8$$

Proposition (3. 29)

Every fuz. c-E-ideal in a KU-involutory algebra Y is fuz. c-ideal .

Proof :

Let $\varphi_{\mathcal{A}}$ be fuz. c-E-ideal in Y and \mathcal{A} is c-E-ideal of Y .

Then \mathcal{A} is c-ideal of Y (By Prop. (2.15)) .

Since $\varphi_{\mathcal{A}}$ is fuz. c-E-ideal of Y , by Definition (3.20) , we have :

$$(1) \forall r \in Y, \varphi_{\mathcal{A}}(\text{constant } 0) \geq \varphi_{\mathcal{A}}(r)$$

$$\begin{aligned} (2) \forall s \in \mathcal{A}, \varphi_{\mathcal{A}}(r) &= \varphi_{\mathcal{A}}(r^{\circ\circ}) \\ &\geq \min\{\varphi_{\mathcal{A}}(r^{\circ} \circ s^{\circ}), \varphi_{\mathcal{A}}(s)\} \\ &= \min\{\varphi_{\mathcal{A}}(s \circ r^{\circ\circ}), \varphi_{\mathcal{A}}(s)\} \quad (\text{ By Prop. (2.5) (2) }) \\ &= \min\{\varphi_{\mathcal{A}}(s \circ r), \varphi_{\mathcal{A}}(s)\} \\ &\quad (\text{ Since } Y \text{ is KU-involutory algebra) .} \end{aligned}$$

Thus $\varphi_{\mathcal{A}}$ is fuz. c-ideal of Y .

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